WEIGHTED SOBOLEV SPACES ON METRIC MEASURE SPACES

LUIGI AMBROSIO, ANDREA PINAMONTI, GARETH SPEIGHT

ABSTRACT. We investigate weighted Sobolev spaces on metric measure spaces $(X, \mathrm{d}, \mathfrak{m})$. Denoting by ρ the weight function, we compare the space $W^{1,p}(X, \mathrm{d}, \rho\mathfrak{m})$ (which always concides with the closure $H^{1,p}(X, \mathrm{d}, \rho\mathfrak{m})$ of Lipschitz functions) with the weighted Sobolev spaces $W^{1,p}_\rho(X, \mathrm{d}, \mathfrak{m})$ and $H^{1,p}_\rho(X, \mathrm{d}, \mathfrak{m})$ defined as in the Euclidean theory of weighted Sobolev spaces. Under mild assumptions on the metric measure structure and on the weight we show that $W^{1,p}(X, \mathrm{d}, \rho\mathfrak{m}) = H^{1,p}_\rho(X, \mathrm{d}, \mathfrak{m})$. We also adapt the results proved by Muckenhoupt and the ones proved by Zhikov to the metric measure setting, considering appropriate conditions on ρ that ensure the equality $W^{1,p}_\rho(X, \mathrm{d}, \mathfrak{m}) = H^{1,p}_\rho(X, \mathrm{d}, \mathfrak{m})$

Scuola Normale Superiore, Piazza dei Cavalieri, 7 - 56126 Pisa,,
E-mail address: `l.ambrosio@sns.it, pinamonti@sns.it, gareth.speight@sns.it`