19th Internet Seminar- Phase 3 (30 May-4 June 2016)
General Ornstein-Uhlenbeck operators

Chiara Spina *

In this project we would like to present some aspects of the theory of semigroups generated by certain second order elliptic operators with unbounded coefficients. In particular, we will focus our attention on the Ornstein-Uhlenbeck operator, the prototype of elliptic operators with unbounded coefficients. Such operator is defined by

$$L = \sum_{i,j=1}^{N} q_{ij} D_{ij} + \sum_{i,j=1}^{N} b_{ij} x_j D_i = \mathrm{tr}(QD^2) + \langle Bx, \nabla \rangle, \qquad x \in \mathbb{R}^N,$$

where $Q = (q_{ij})_{i,j=1,\dots,N}$ is a real symmetric and positive definite matrix and $B = (b_{ij})_{i,j=1,\dots,N}$ is a non-zero real matrix.

We will start by illustrating the heuristic argument applied by Kolmogorov to get an explicit representation formula for the generated semigroup $T(t)$. This formula allows the study of the main properties of the semigroup (see [4] and [1, Section 3]).

Then we will show some classical results for the Ornstein-Uhlenbech operator in continuous function spaces and in L^p spaces with respect to Lebesgue measure. Especially we will consider generation properties of suitable realizations of L, strong continuity, contractivity, smoothing effects of $(T(t))_{t\geq 0}$ ([2], [3], [6]).

As further result we could present the spectral properties of the Ornstein Uhlenbeck operator in $L^p(\mathbb{R}^N)$ with respect to the Lebesgue measure ([5]).

References

[1] L. Angiuli, G. Metafune, C. Spina, *Feller semigroups and invariant measures*, Riv Mat. Univ. Parma, Vol. 1 (2010), 347-406.

[2] G. Da Prato, A. Lunardi, *On the Ornstein-Uhlenbeck Operator in Spaces of Continuous Functions*, J. Funct. Anal., **1**(1995), 94-114.

[3] A. Lunardi, V. Vespri, *Generation of strongly continuous semigroups by elliptic operators with unbounded coefficients in* $L^p(\mathbb{R}^N)$, Rend. Istit. Mat. Univ. Trieste **28** (1997), 251-279.

[4] A. N. Kolmogorov, *Zufallige Bewegungen,* Ann. of Math. **116**, 2, 35 (1934), 116-117.

*Dipartimento di Matematica e Fisica "Ennio De Giorgi", University of Salento, C.P.193, 73100, Lecce, Italy. e-mail: chiara.spina@unisalento.it

[5] G. Metafune, *L^p-spectrum of Ornstein-Uhlenbeck operators*, Ann. Sc. Norm. Sup. Pisa **30** (2001), 97–124.

[6] G. Metafune, D. Pallara, M. Wacker, *Feller semigroups on $\mathbb{R}^N$,* Semigroup Forum **65** (2002), 159-205.