Project

The Ornstein-Uhlenbeck operator is presented in Lectures 12 ,13 and 14 as the generator of the following semigroup with same name : $T_t(f)(x) = \int_X f(e^{-t}x + \sqrt{1-e^{-2t}}y)\gamma(dy)$. Its domain was studied and it was characterized in various subspaces of $L^2(X)$, where X denotes a separable Banach space, mainly in $\mathbf{C_b}(\mathbf{X})$ and $\mathbf{C^1_b}(\mathbf{X})$. Its spectrum was well determined even in infinite dimensional case.

We propose to give developments on alternative methods to define the Ornstein-Uhlenbeck operator on $L^p(X)$ when X is an infinite dimensional separable Banach space, mainly for $p \neq 2$. To be more precise, first we define the operator

$$S_t(f)(x) = \int_X f(x + \sqrt{t}y)\gamma(dy).$$

The aim of the project is to prove that $S(t)_{t\geq 0}$ is a semigroup with generator related to a realization of the Laplacian on a convenient subspace of X. Then we obtain the Ornstein-Uhlenbeck operator as a sum of two operators : the Lapalcian and a suitable multiplication by gradient. At the second step, we give a stochastic aspect of the Ornstein-Uhlenbeck process and highlight its importance in the resolution of the associate stochastic equation. The finality will be to establish a bridge between the analytic point of view and the probabilistic one.

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