H-Lipschitz functions

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Abstract

The notion of Lipschitz function in finite dimensional spaces is classical and it is well-known that the class of bounded Lipschitz continuous functions in $\mathbb{R}^d$ coincides with the Sobolev space $W^{1,\infty}(\mathbb{R}^d)$ of L^∞ weakly differentiable functions with L^∞-derivative. Another important property of Lipschitz functions is that they are a.e. differentiable (Rademacher theorem). All these results are true both with the Lebesgue and the standard Gaussian measure on $\mathbb{R}^d$. In the Gaussian case, it is also true that $W^{1,\infty}(\mathbb{R}^d, \gamma_d) \subset W^{1,p}(\mathbb{R}^d, \gamma_d)$ for all $p \geq 1$. The aim of the project is to consider all these issues in the infinite dimensional case. This is a natural complement of the theory of Sobolev spaces, as in the lectures the space $W^{1,\infty}(X, \gamma)$ has not been defined. Here of course we consider H-Lipschitz functions, i.e., measurable functions $f : X \to \mathbb{R}$ such that there is $L > 0$ such that

$$|f(x+h) - f(x)| \leq L|h|_H \qquad \forall\, h \in H, \quad \gamma - a.e.\ x \in X,$$

where as usual H is the Cameron-Martin space. Possible further issues can be vector-valued H-Lipschitz functions and the extension of a Lipschitz function defined on a subset of X, which turns out to be nontrivial even in finite dimensions, when vector-valued functions are considered.

All the relevant material is contained in Sections 4.5 and 5.11 of [1].

References

[1] V. I. Bogachev: *Gaussian Measures.* American Mathematical Society, 1998.