KOLMOGOROV EQUATIONS PERTURBED BY AN INVERSE-SQUARE POTENTIAL

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It is known that, if A is a second order elliptic operator acting on real functions defined on a domain in $\mathbb{R}^N$ and if V is a positive function which is sufficiently singular, then the problem

$$\frac{\partial u}{\partial t} = Au + V(x)u$$

may not have any positive solutions. For example, consider

$$\frac{\partial u}{\partial t} = \Delta u + \frac{c}{|x|^2}u; \quad t > 0, \quad x \in \mathbb{R}^N. \tag{1}$$

In 1984, P. Baras and J. Goldstein [1] showed that (1) has many positive solutions if $c \leq \left(\frac{N-2}{2}\right)^2$ but no positive solutions at all in the sense of distributions if $c > \left(\frac{N-2}{2}\right)^2$.

In this project, we will focus on new results of this type obtained recently by Goldstein and Rhandi [4, 5] replacing the Laplacian by the Kolmogorov operator

$$L_\rho u = \Delta u + \frac{\nabla \rho}{\rho} \cdot \nabla u.$$

Here $\rho : \mathbb{R}^N \to (0, \infty)$ is a sufficiently smooth probability density.

References

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