Margherita Piazzolla Beloch (1879-1976)

The geometry of paper folding and the resolution of problems of third degree

A glimpse on Margherita Piazzolla Beloch's life

- 1879 Born in Frascati (Rome)
- 1908 graduation in Mathematics, in Rome University
- 1927 full professor of Geometry in Ferrara
- 1938 prize for the Inventions Exhibition Leonardo Da Vinci
- 1949 retirement
- 1953 book Lessons of Complementary Mathematics - *Elementary Mathematics from a Higher Standpoint*

Margherita was interested in algebraic geometry, topology, but also aerophotogrammetry and röntgenphotogrammetry

Last, but non least, she gave important contributions on didactics of mathematics.

Complementary mathematics:
course founded in Italy
to prepare prospective high school mathematics teachers


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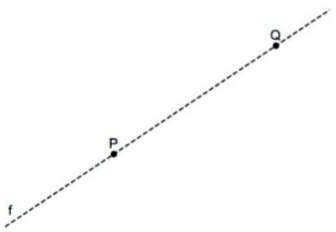
LEZIONI
DI
MATEMATICA COMPLEMENTARE
(La Matematica Elementare vista dall'alto)
redatte dal Prof. EGIDIO ORZALESI

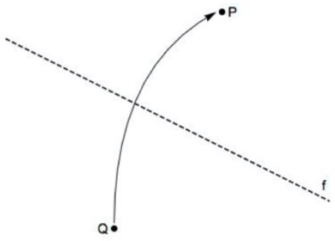
Pubblicazioni dell'Istituto di Geometria
dell'Università di Ferrara
1953

Lessons of Complementary Mathematics (Elementary mathematics from a higher standpoint)

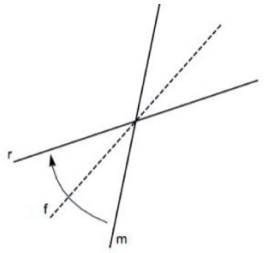
Sundara Row *Geometric exercises in paper folding*, Addison and Co., Madras, 1893.
Pictures of the covers collected in
P. Magrone, V. Talamanca, (2017)



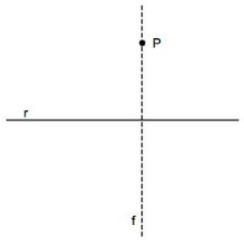
(O1) we can fold the line through two points.



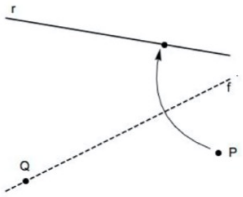
(O2) we can fold the median of the segment joining two points.



(O3) we can fold the bisector of an angle.

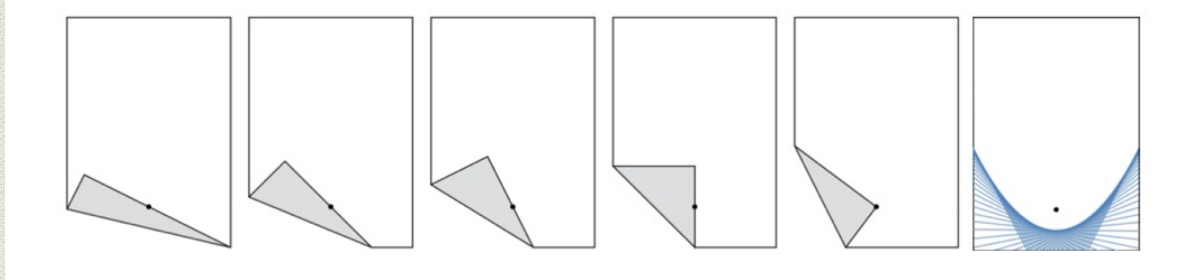


(O4) we can fold the line perpendicular to a given line passing through a given point.

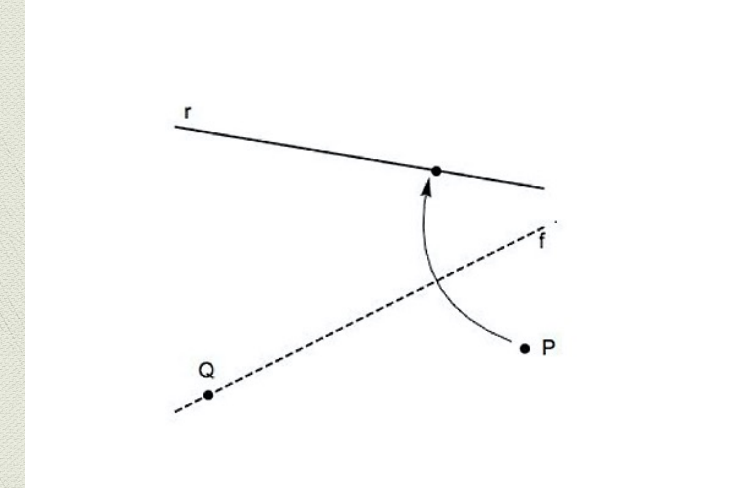


(O5) Given two point P and Q and a line r, we can fold the line passing through Q and reflecting P onto r.

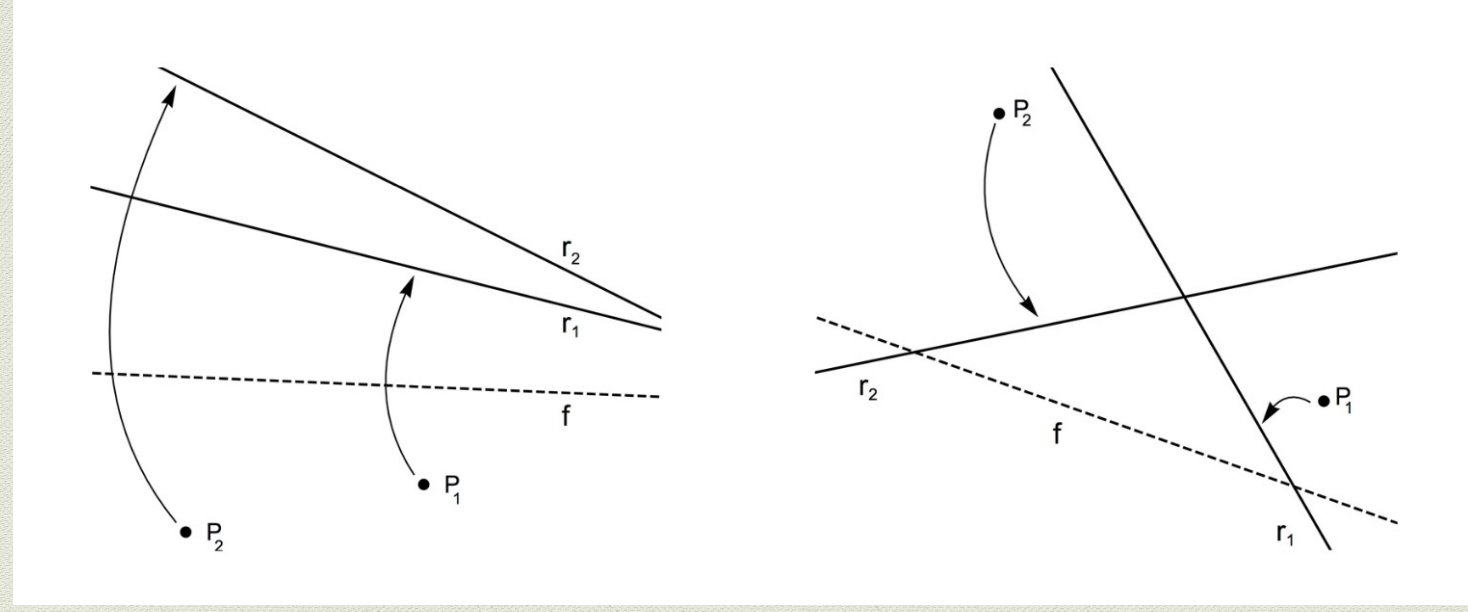
geometric interpretation
of fold O5



the line r is the edge of the paper



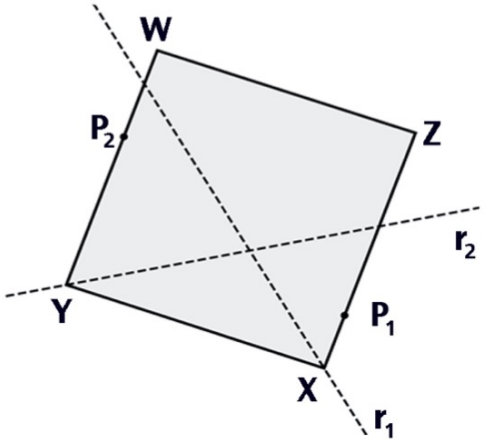
Beloch's fold



Two different positions for performing Beloch's fold

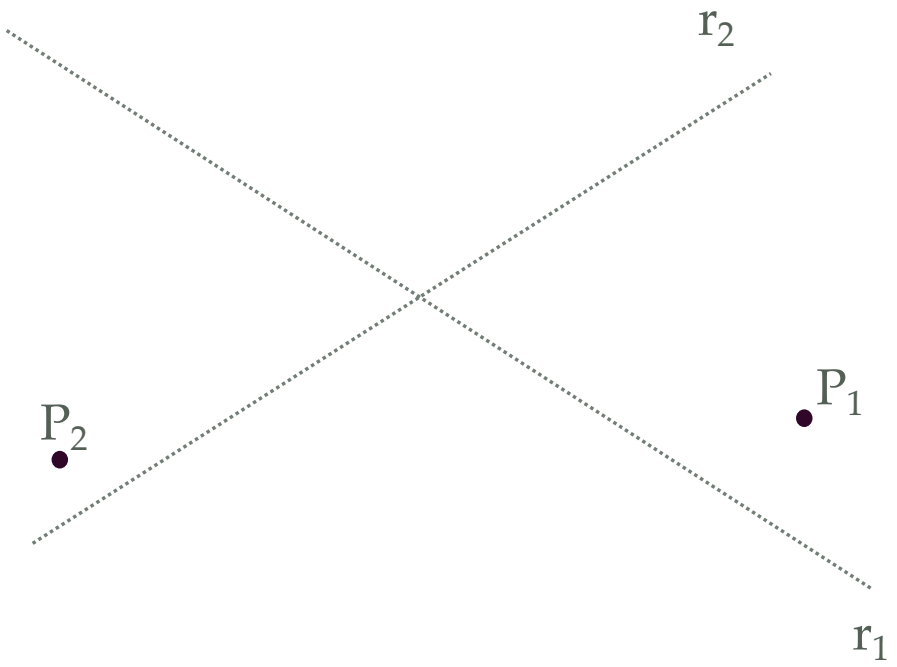
Given two point P_1 and P_2 and two lines r_1 and r_2 then, when it exists, we can fold the line reflecting P_1 onto r_1 and P_2 onto r_2.

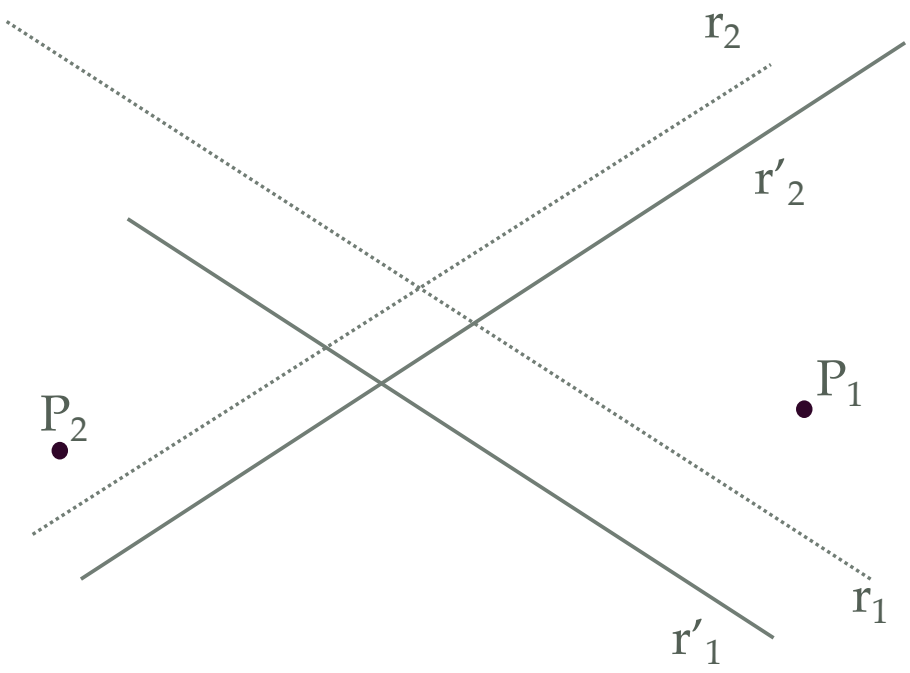
Costruire un quadrato di cui due lati opposti passino rispettivamente per due punti dati, e i due rimanenti vertici situati sui rimanenti lati stiano rispettivamente su due rette date

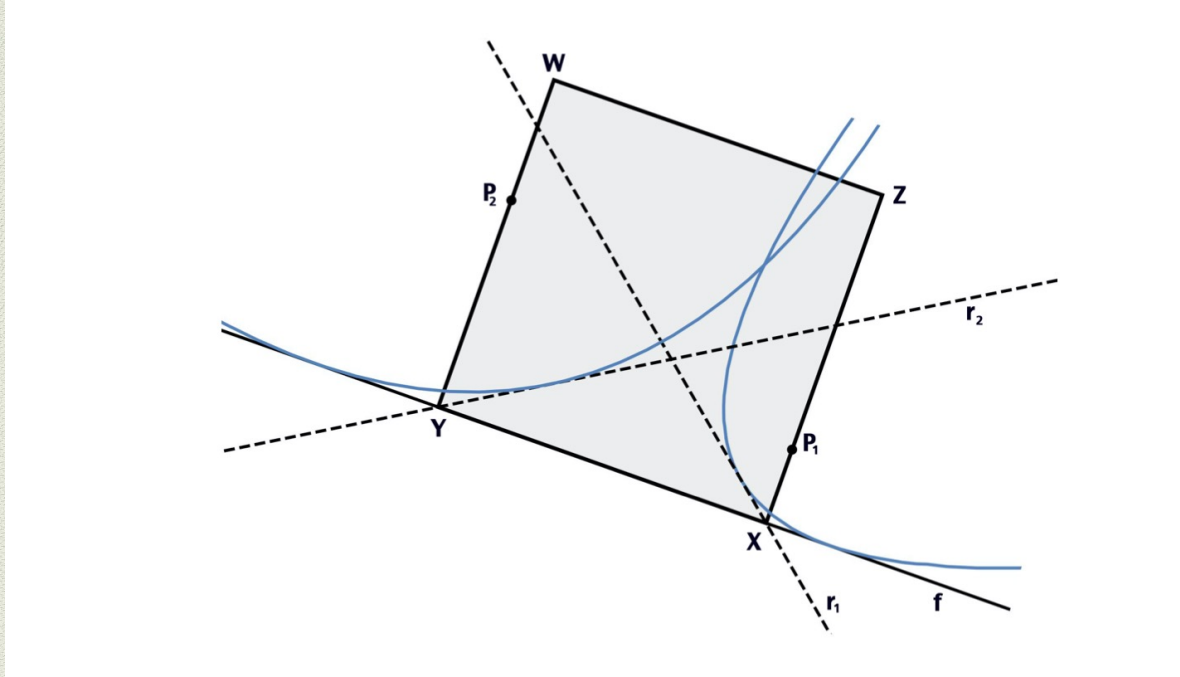


Given two points, P_1 and P_2, and two lines, r_1 and r_2, construct a square with two opposite edges passing through the two given points, and the remaining vertices lying on the two given lines

The square construction is the key to construct solutions of third degree problems by paper folding. How is Beloch's fold connected to this square?


r2
P1
P2
r1


r2
r′2
P1
P2
r1
r′1


W
Z
P2
r2
Y
P1
X
r1
f

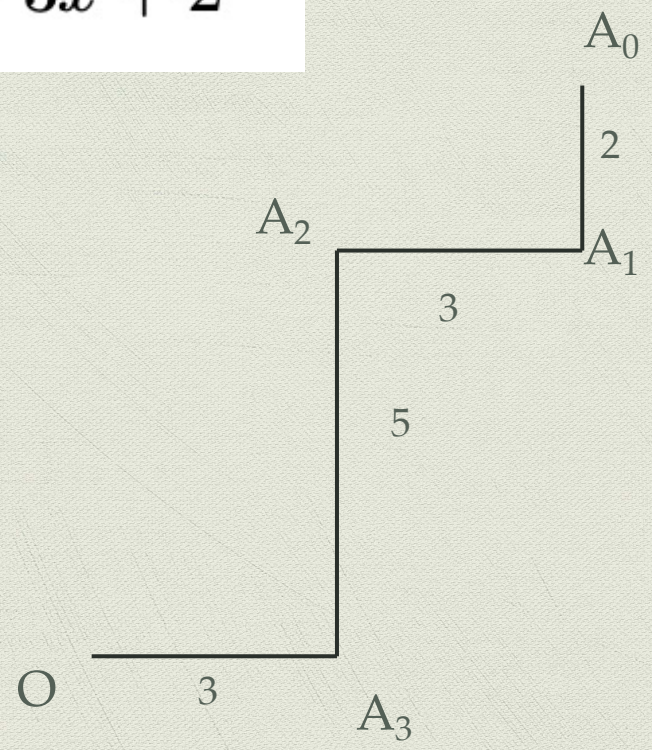
A few words about Lill's method

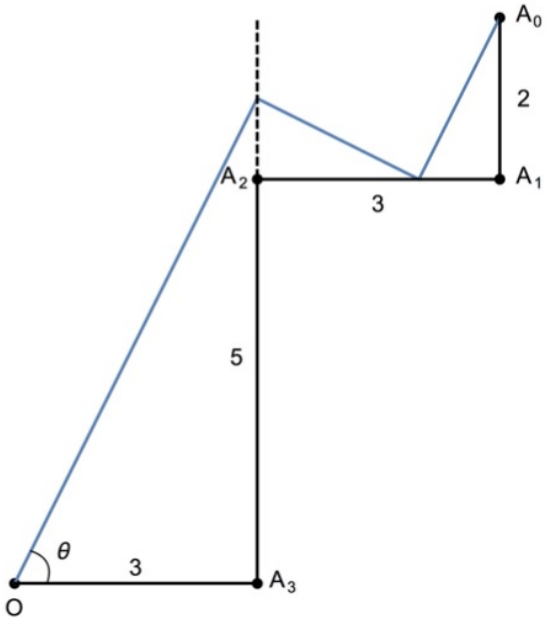
$P(X) = a_3x^3 + a_2x^2 + a_1x + a_0$

4 edge polygonal chain O-A3-A2-A1-A0, with only right angles, where the starting point is the origin O, the length of the segment ending in Ai is $|a_i|$ and in Ai the turn is

clockwise if $a_i a_{i-1} \geq 0$ and counterclockwise if $a_i a_{i-1} \leq 0$.

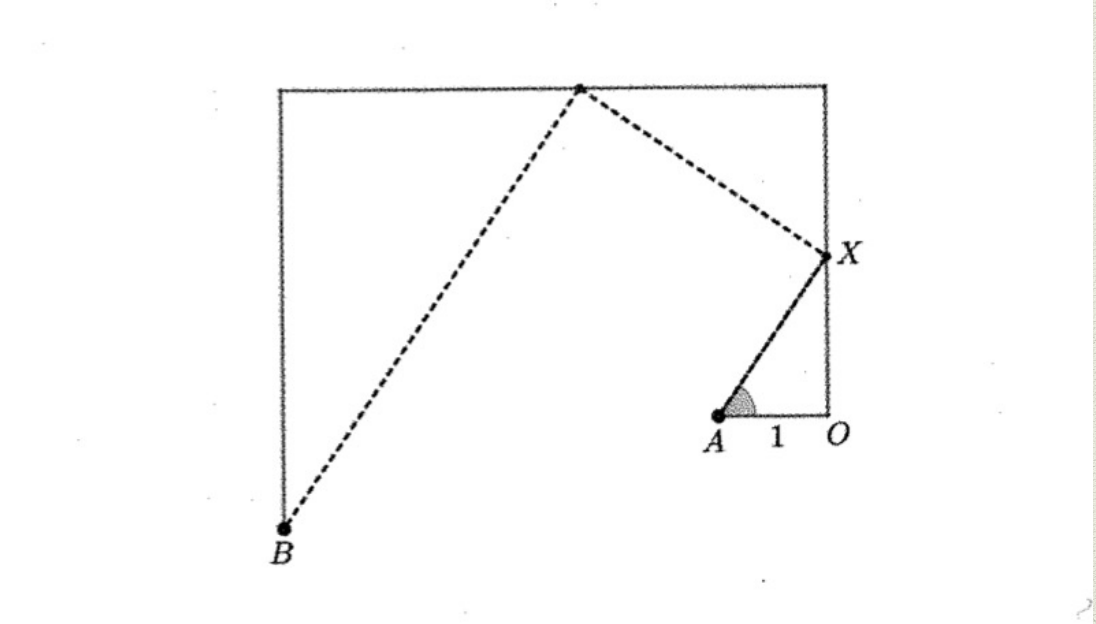
P(x)= $3x^3 - 5x^2 - 3x + 2$



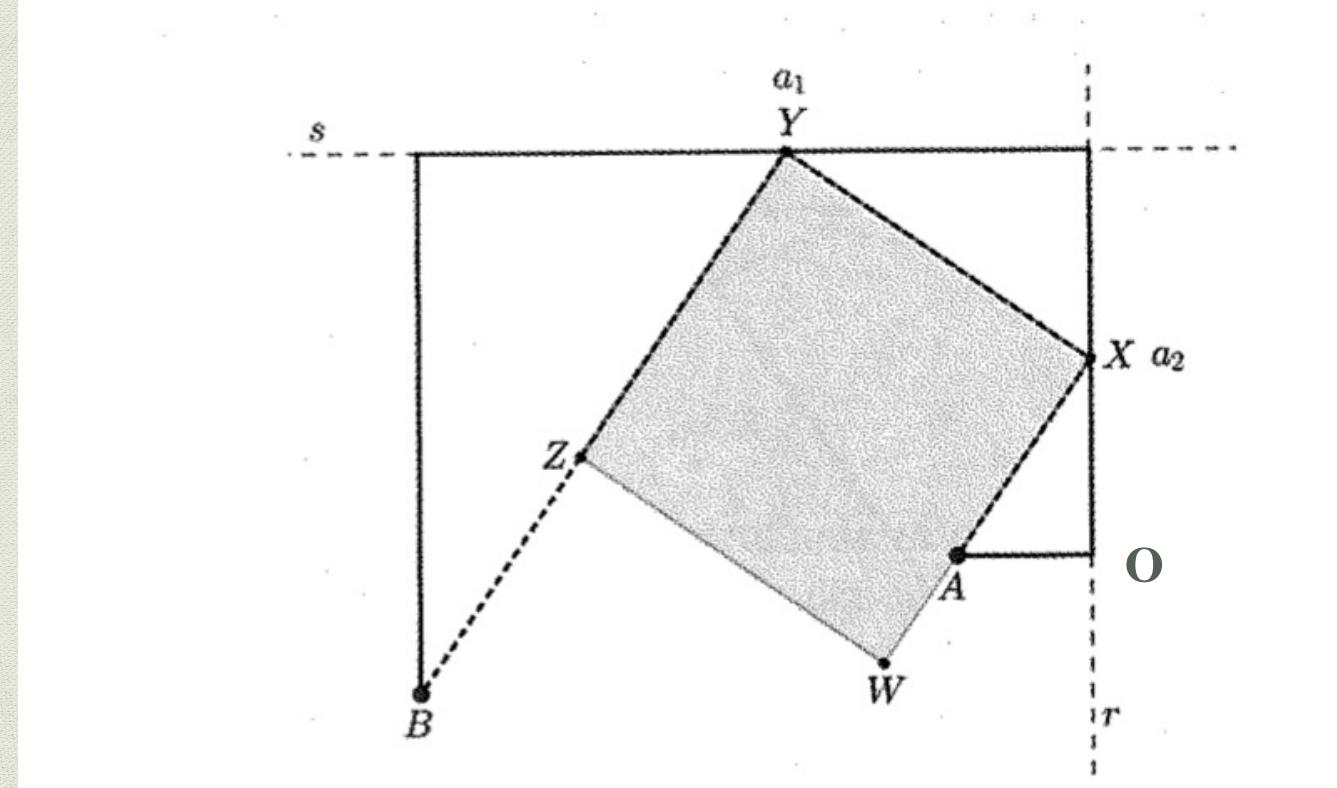


a $\tan(\theta) = 2$ is a root of $3x^3 - 5x^2 - 3x + 2$

How is Lill's method connected with Beloch's square



Picture from Borgato Salmi 2018



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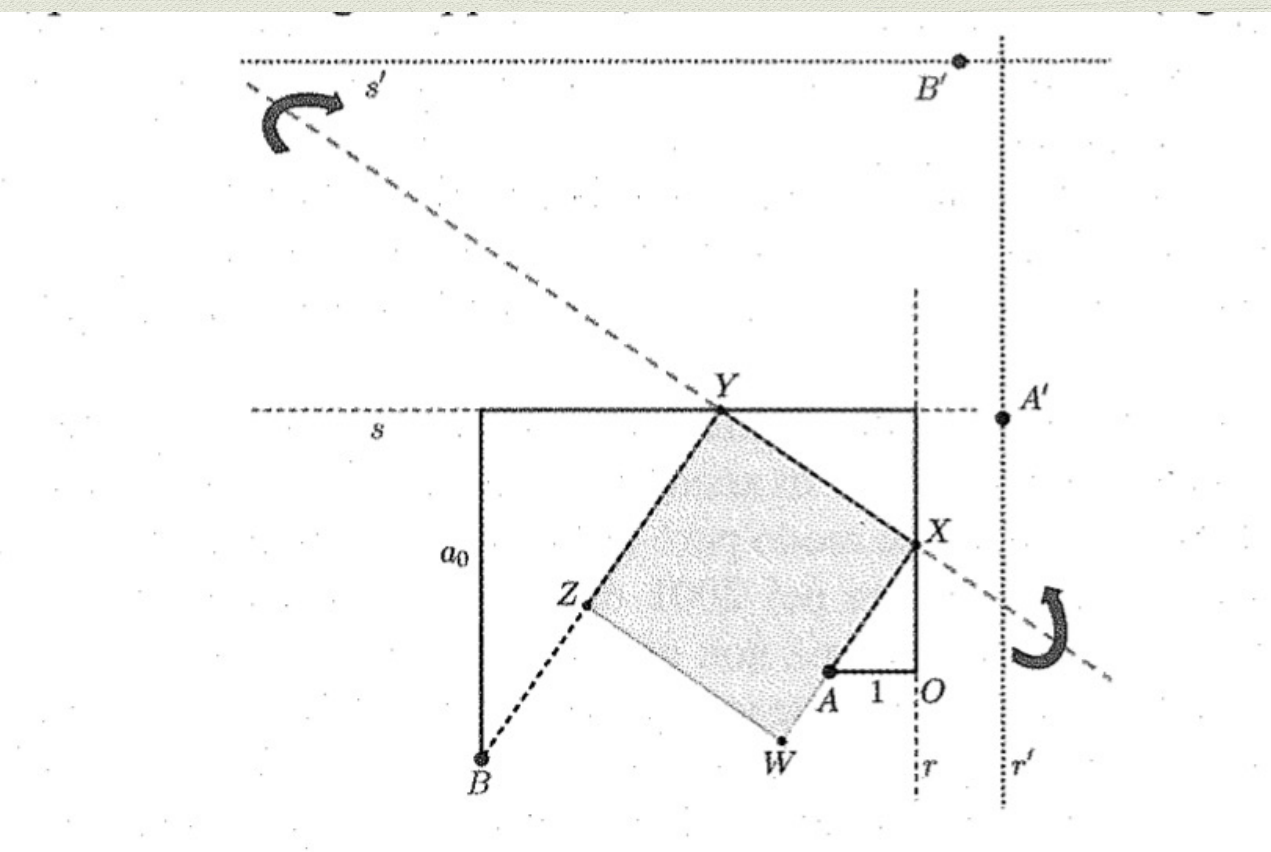

Picture from Borgato Salmi 2018

References

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