The Systems of the World by Riccioli

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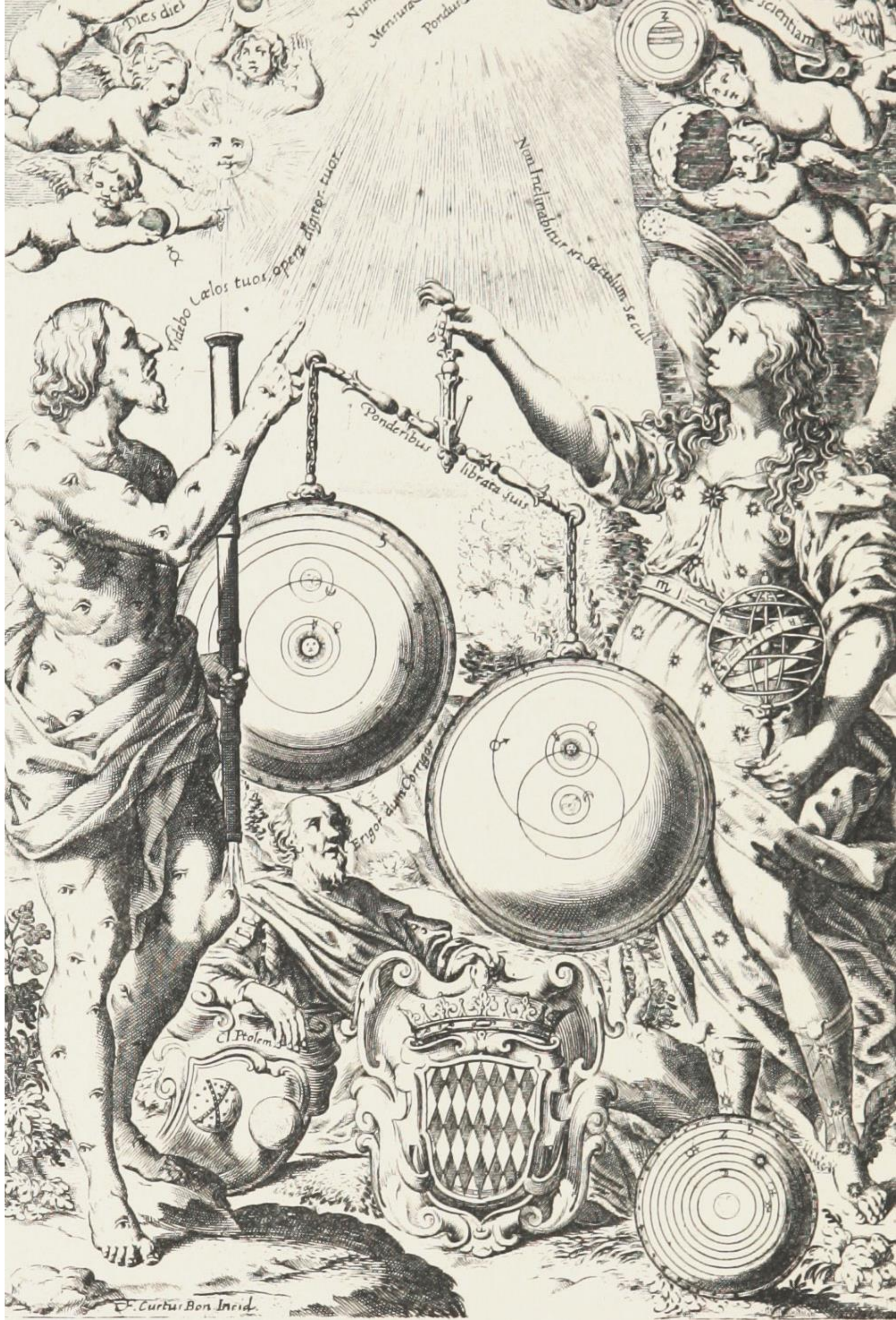
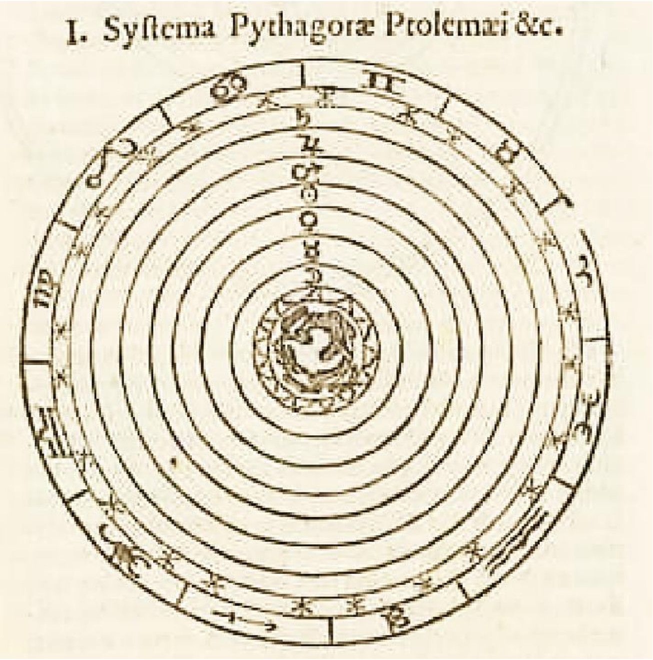
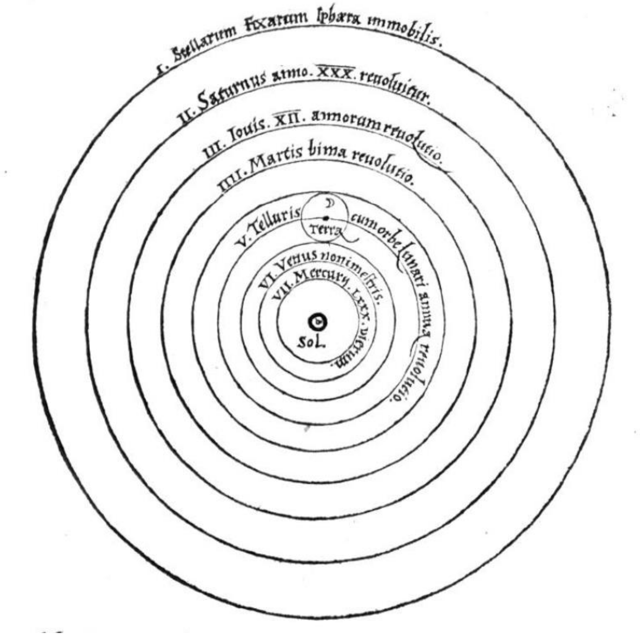


Introduction

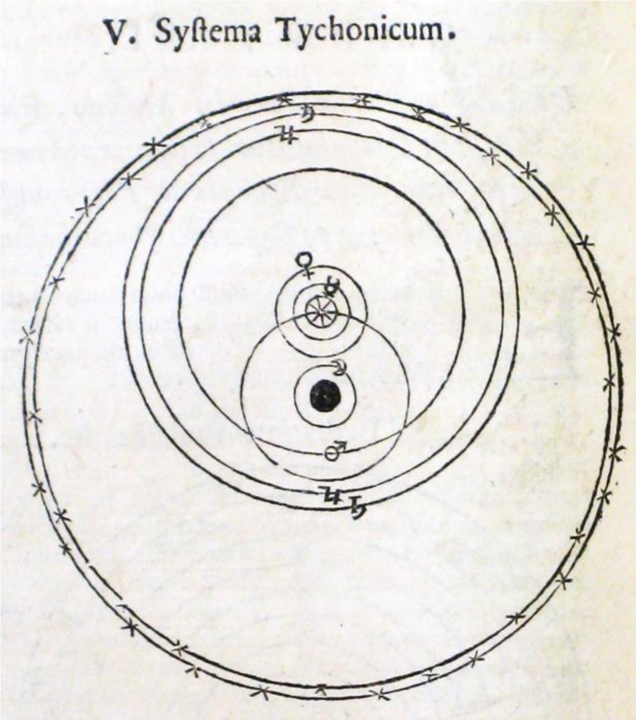
Ptolemaic system



Copernican system

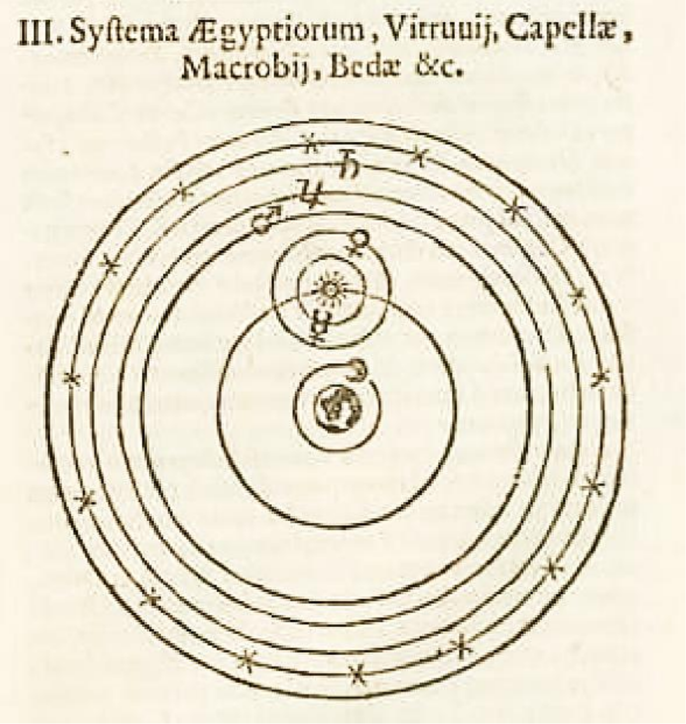


Ticonic system

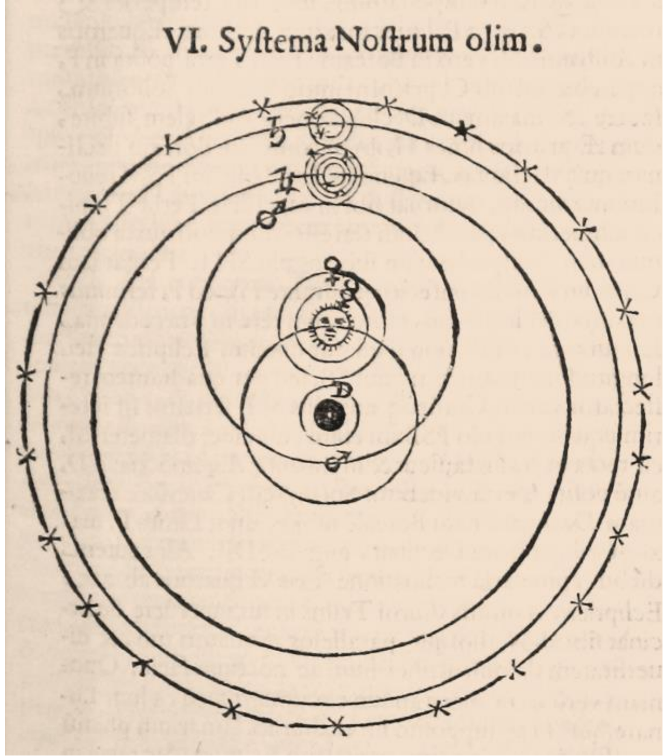


Introduction

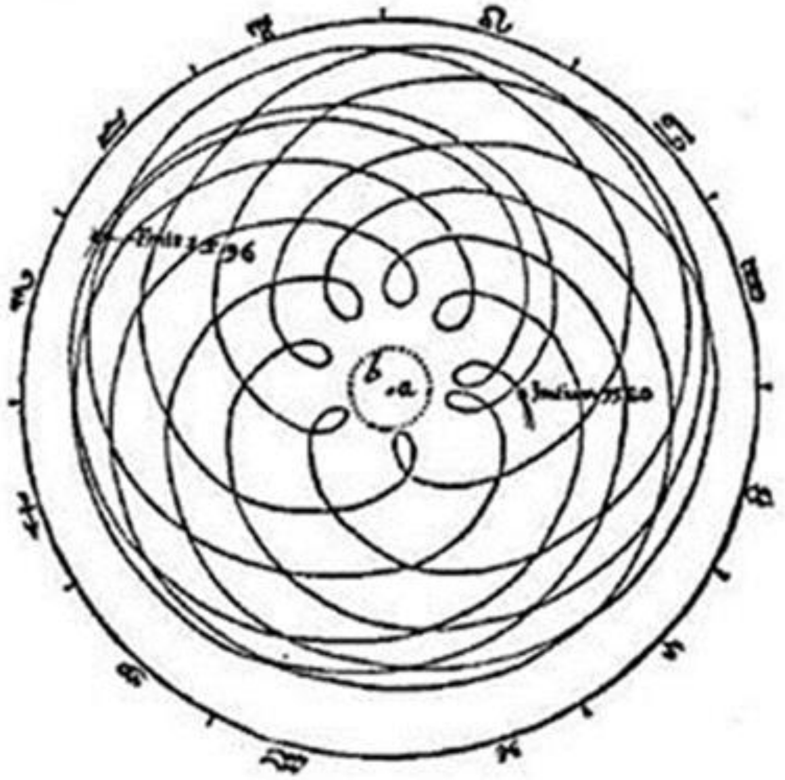
Cappella system



Riccioli system



Introduction



Trajectory of Mars

The model was drawn by Kepler using Tycho's data. The earth is stationary and is located at the center of the universe.

The educational pathway for in-depth study

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Feature

Topic

Hypocycloids, epicycloids, hypothrocoids and epitrocoides

Target audience

Second biennium of high school

Timelines

2 hours

Tools

Photocopy, dynamic geometry software

Methodology

Peer education, teaching laboratory

Feature

Indicazioni Nazionali (2010), Specific learning goals

Physics: “The study of gravitation [...] will enable the student to acquire a deeper knowledge of the 16th- and 17th-century debate on cosmological systems within a historical and philosophical framework.”

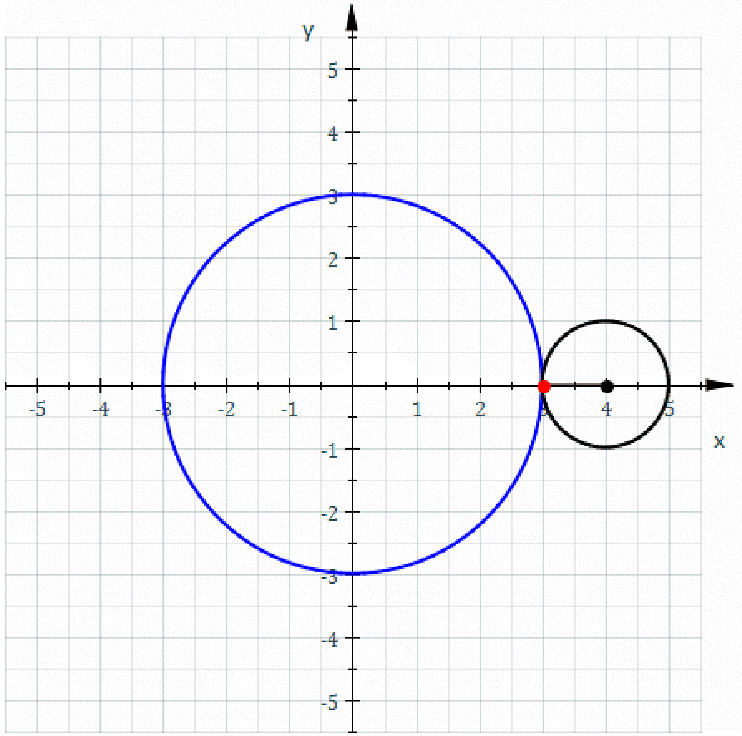
Philosophy: “Regarding modern philosophy, essential themes and authors will be: the scientific revolution and Galilei, [...].”

History: “It is appropriate that some crucial themes (for example: the birth of scientific culture in the seventeenth century, [...]) be treated in an interdisciplinary way, in relation to the other teachings.”

Lesson content (Epicycloids)

Definition.

The ***epicycloid*** is the curve described by a fixed point on the circumference of a circle as it rolls on the outside of the circumference of a fixed circle.



Lesson content (Epicycloids)

Epicicloide 1 – GeoGebra

Epicicloide 2 - GeoGebra

Vary the sliders R and r, which represent the radius of the base circle and epicycle, respectively, and answer the following questions. If the ratio n between the radii R and r is:

- a natural number, the curve is ____

- a rational number, the curve is ____

Although you cannot experiment with it in GeoGebra, what do you think will happen if n is irrational?

____

Lesson content (Epicycloids)

Stimulus questions

1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

Lesson content (Epicycloids)

Stimulus questions

1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

If n is a natural number:

1. Closed
2. Unbraided
3. n
4. 1

Lesson content (Epicycloids)

Stimulus questions

1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

If $n = a/b$ rational number:

1. Closed
2. Braided
3. a
4. b

Lesson content (Epicycloids)

Property.

If the ratio *n* between the radii of the base circle and the epicycle is:

- a natural number, the curve is closed and unbraided and has n cusps (trajectory is periodic);
- a rational number, the curve is closed and braided (trajectory is periodic);
- an irrational number, the curve is open (trajectory is aperiodic).

Lesson content (Epicycloids)

Discover some special cases of the epicycloid by varying the sliders R and r as required.

- Vary sliders R and r so that $n = 1$. Are you familiar with this curve?

  Try with different values of R and r from before, keeping their ratio constant $n = 1$. What differences are there between the new curve and the previous one?____________________________

- Vary sliders R and r so that $n = 2$. Do you know this curve?

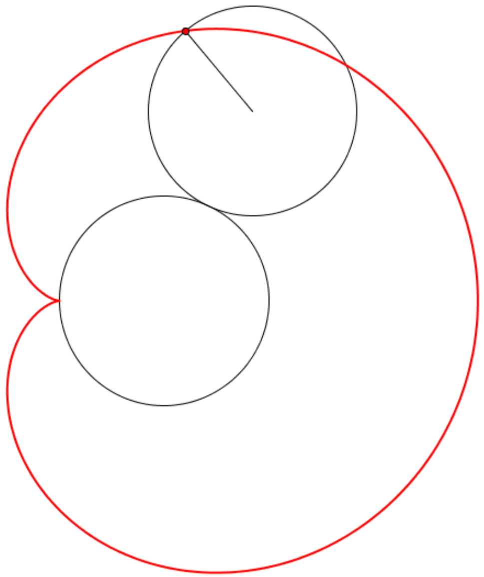
  Try with different values of R and r from before, keeping their ratio constant $n = 2$. What differences are there between the new curve and the previous one?____________________________

Try your own experiment, varying the R and r sliders, and look for other interesting curves. Keep trace of your best successful attempts. ____

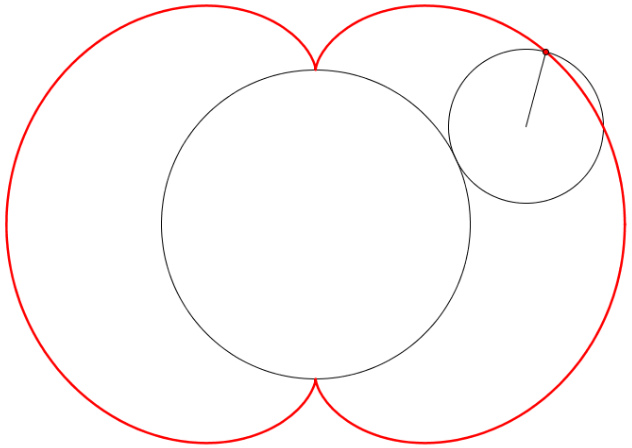
Lesson content (Epicycloids)

Special cases.

- The cardioid is an epicycloid with $n = 1$.



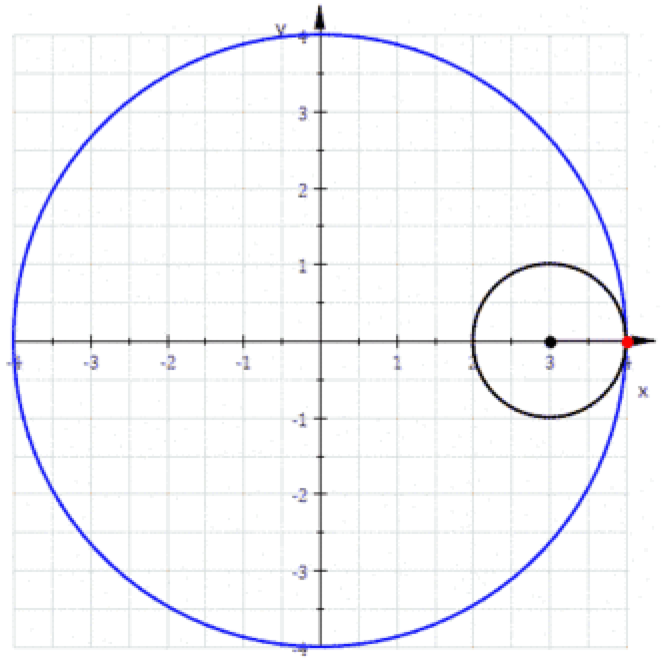
- Nephroid is an epicycloid with $n = 2$.



Lesson content (Hypocycloids)

Definition.

The ***hypocycloid*** is the curve described by a fixed point on the circumference of a circle as it rolls on the inside circumference of a fixed circle.



Lesson content (Hypocycloids)

Ipocicloide 1 – GeoGebra

Ipocicloide 2 – GeoGebra

Vary the sliders R and r, which represent the radius of the base circle and epicycle, respectively, and answer the following questions. If the ratio n between the radii R and r is:

- a natural number, the curve is ______________________________
- a rational number, the curve is ______________________________

Although you cannot experiment with it in GeoGebra, what do you think will happen if n is irrational?

____

Lesson content (Hypocycloids)

Stimulus questions

1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

Lesson content (Hypocycloids)

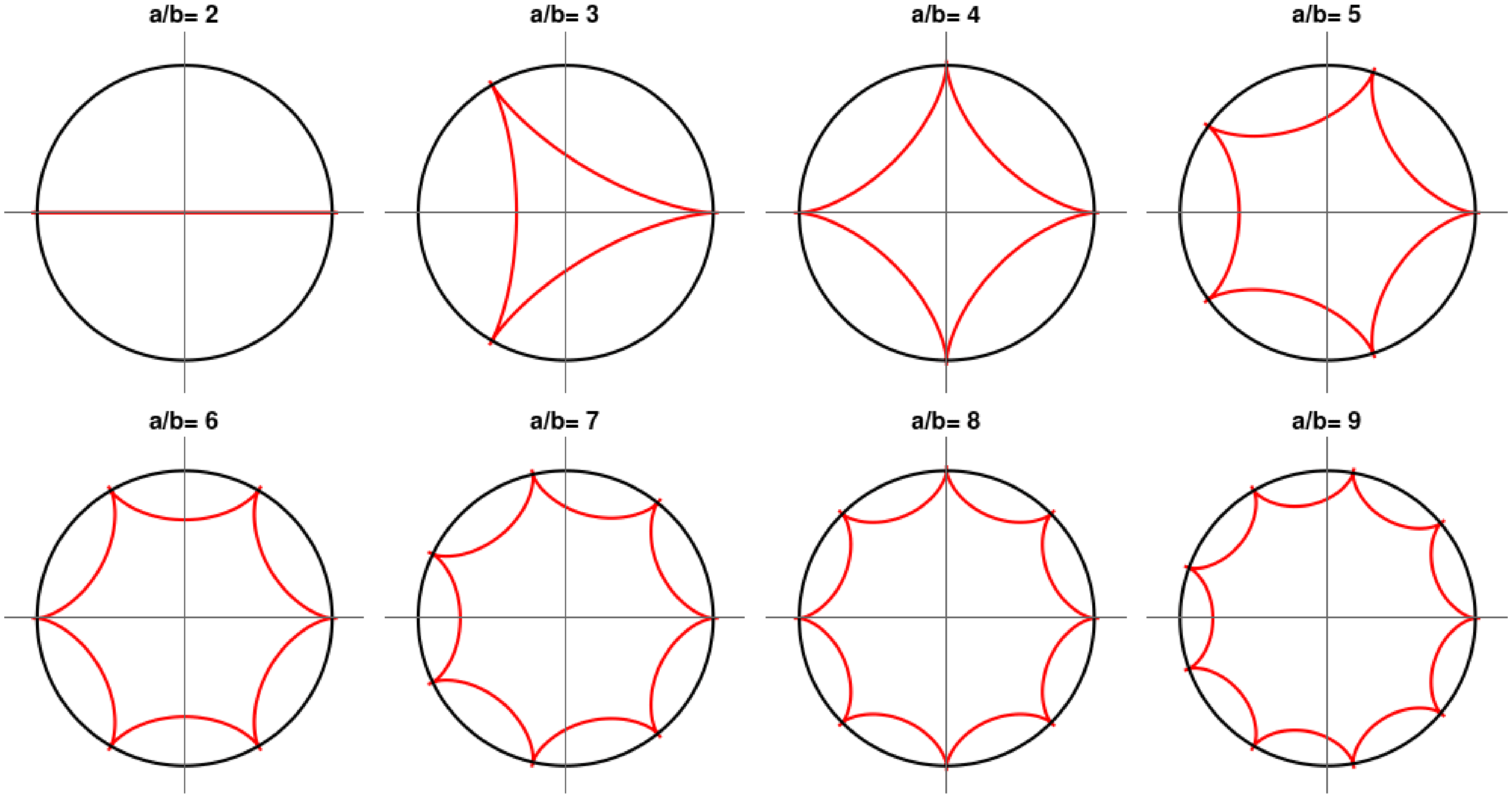
Stimulus questions

1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

If n is a natural number:

1. Closed
2. Unbraided
3. n
4. 1

Lesson content (Hypocycloids)



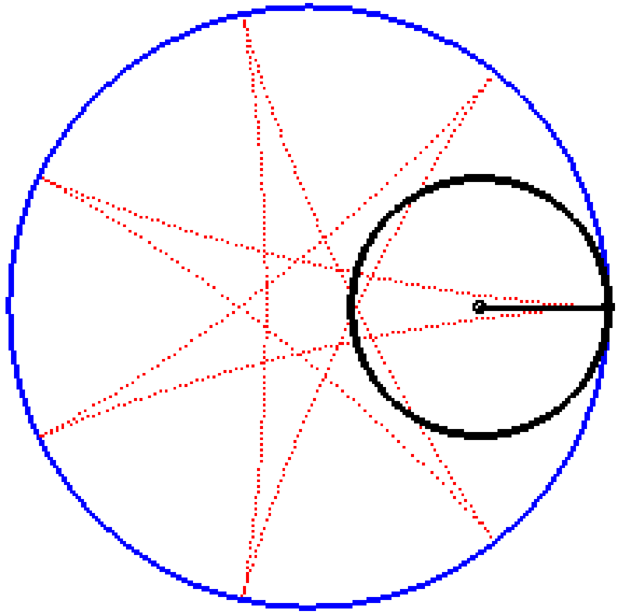
Lesson content (Hypocycloids)

Stimulus questions

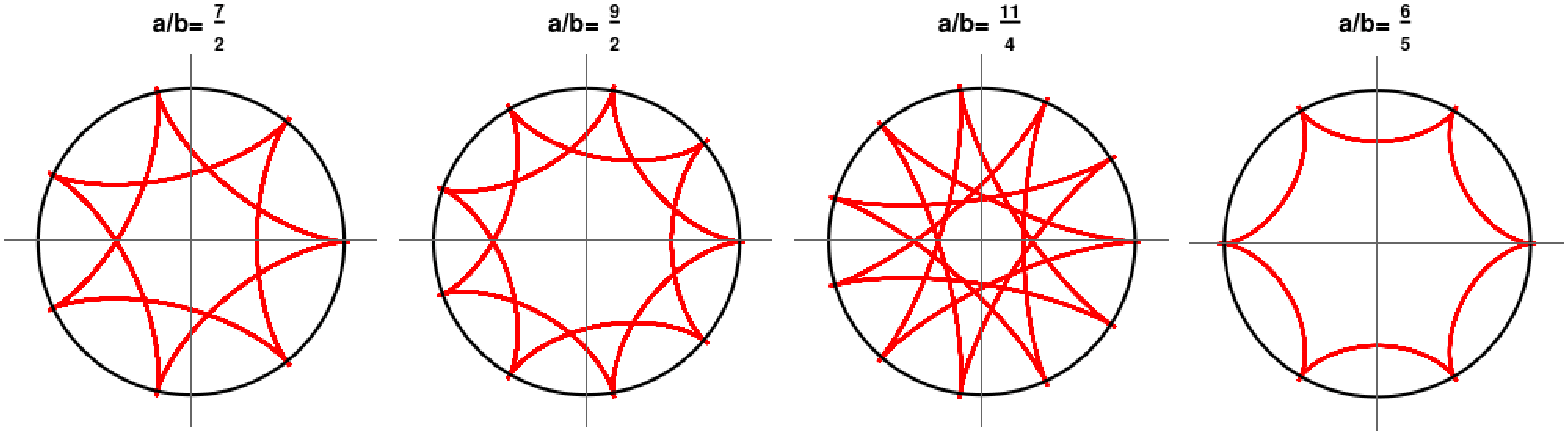
1. Is the curve open or closed?
2. Is the curve braided or unbraided?
3. How many revolutions does the epicycle make around its center to close the curve?
4. How many revolutions of the deferens does the epicycle make to close the curve?

If $n = a/b$ rational number:

1. Closed
2. It depends
3. a
4. b



Lesson content (Hypocycloids)



Lesson content (Hypocycloids)

Property.

If the ratio *n* between the radii of the base circle and the epicycle is:

- a natural number, the curve is closed and corresponds to an unbraided n-pointed star (trajectory is periodic);
- a rational number, the curve is closed and corresponds to a star (trajectory is periodic);
- an irrational number, the curve is open (trajectory is aperiodic).

Lesson content (Hypocycloids)

Discover some special cases of the hypocycloid by varying the sliders R and r as required.

- Vary sliders R and r so that $n = 2$. Are you familiar with this curve?

  Try with different values of R and r from before, keeping their ratio constant $n = 2$. What differences are there between the new curve and the previous one?____________________________

- Vary sliders R and r so that $n = 4$. Do you know this curve?

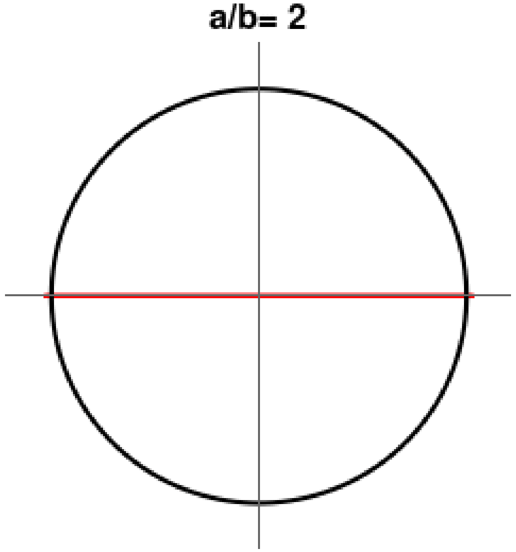
  Try with different values of R and r from before, keeping their ratio constant $n = 4$. What differences are there between the new curve and the previous one?____________________________

Try your own experiment, varying the R and r sliders, and look for other interesting curves. Keep trace of your best successful attempts. ____

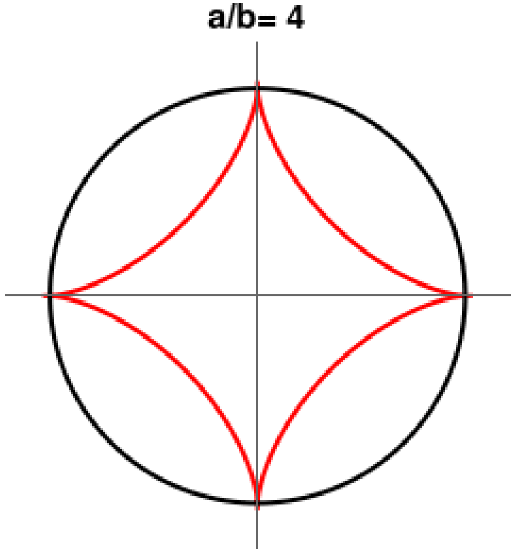
Lesson content (Hypocycloids)

Special cases.

- A hypocycloid with n =2 is a segment (Cardano's theorem).



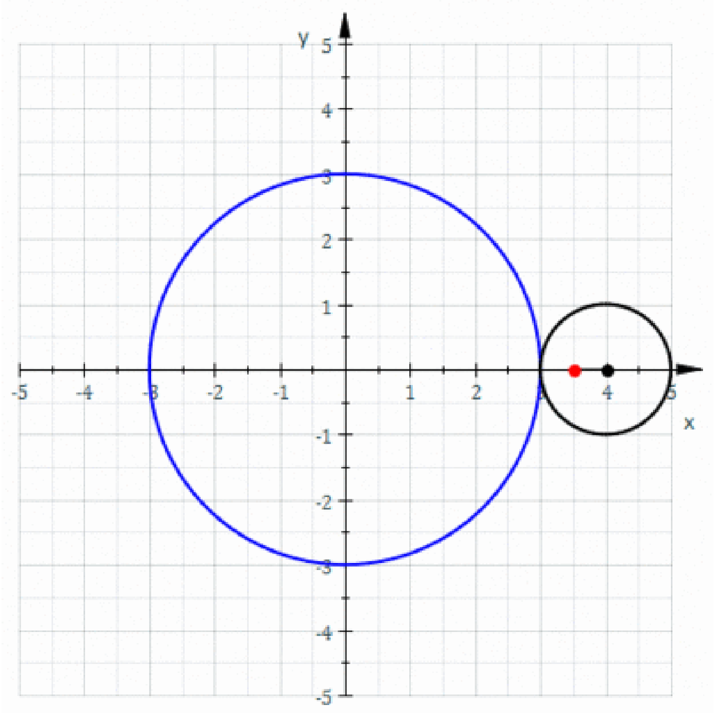
- An asteroid is a hypocycloid with $n = 4$.



Lesson content (Epitrochoid)

Definition.

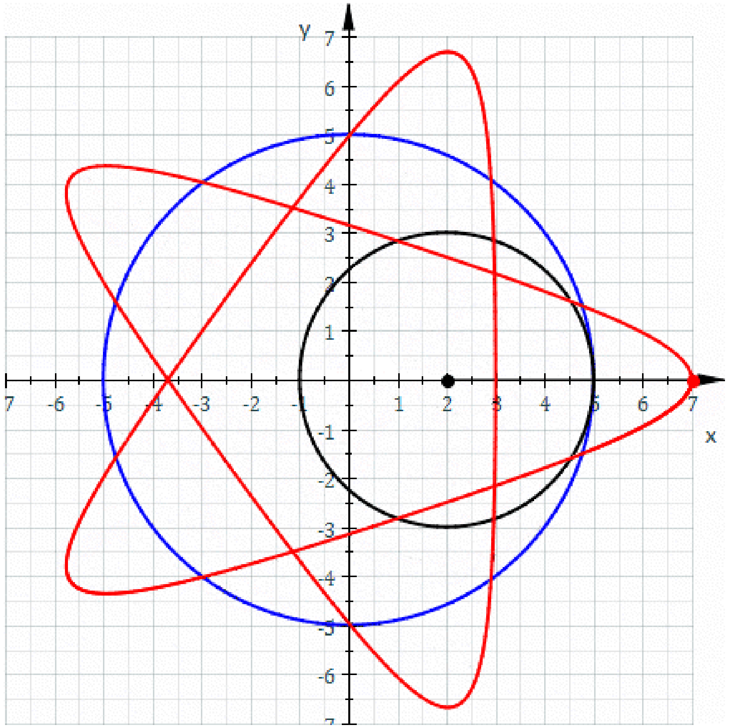
The ***epitrochoid*** is a roulette traced by a point attached to a circle of radius r rolling around the outside of a fixed circle of radius R, where the point is at a distance d from the center of the exterior circle.

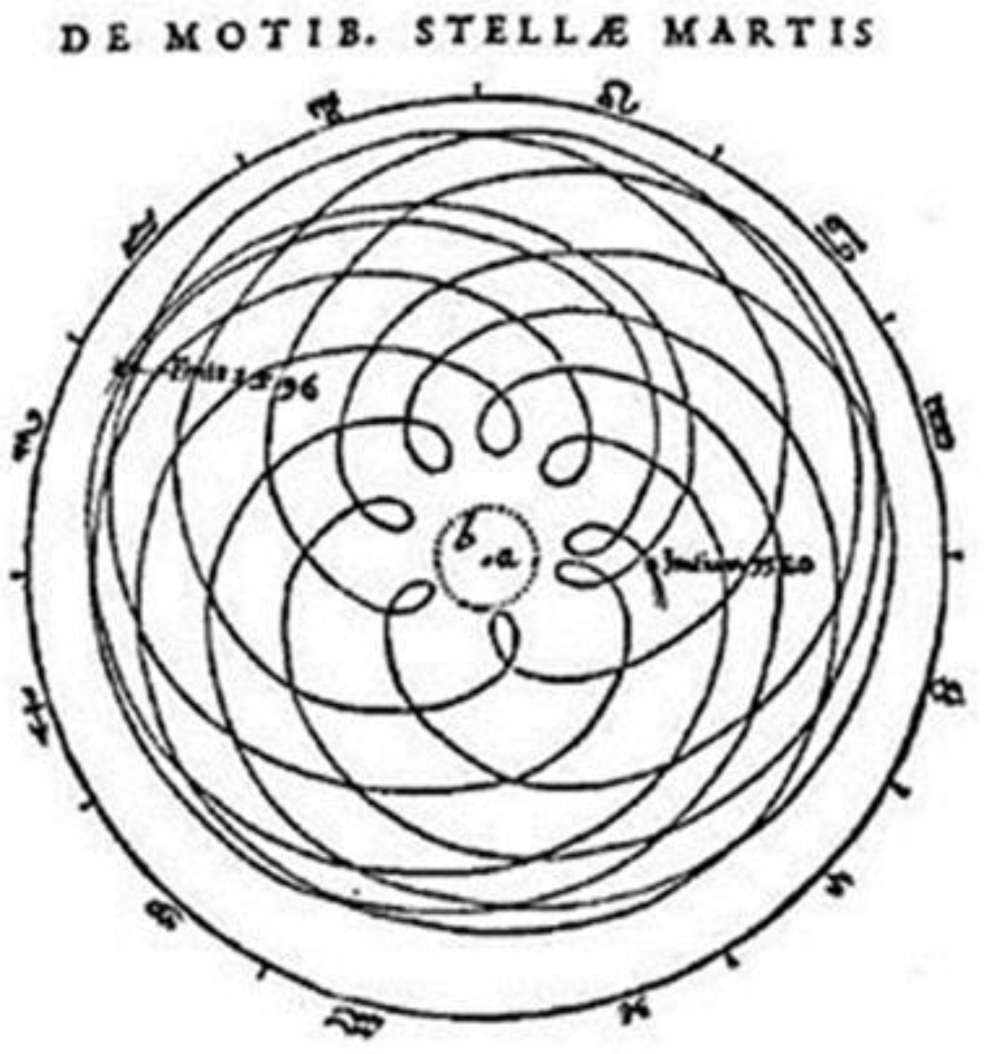


Lesson content (Hypotrochoid)

Definition.

The ***hypotrochoid*** is a roulette traced by a point attached to a circle of radius *r* rolling around the inside of a base circle of radius *R*, where the point is a distance *d* from the center of the interior circle.





THANK YOU FOR YOUR ATTENTION