From Scipione Del Ferro to Rafael Bombelli: the progress of algebra in Italy in the XVI century

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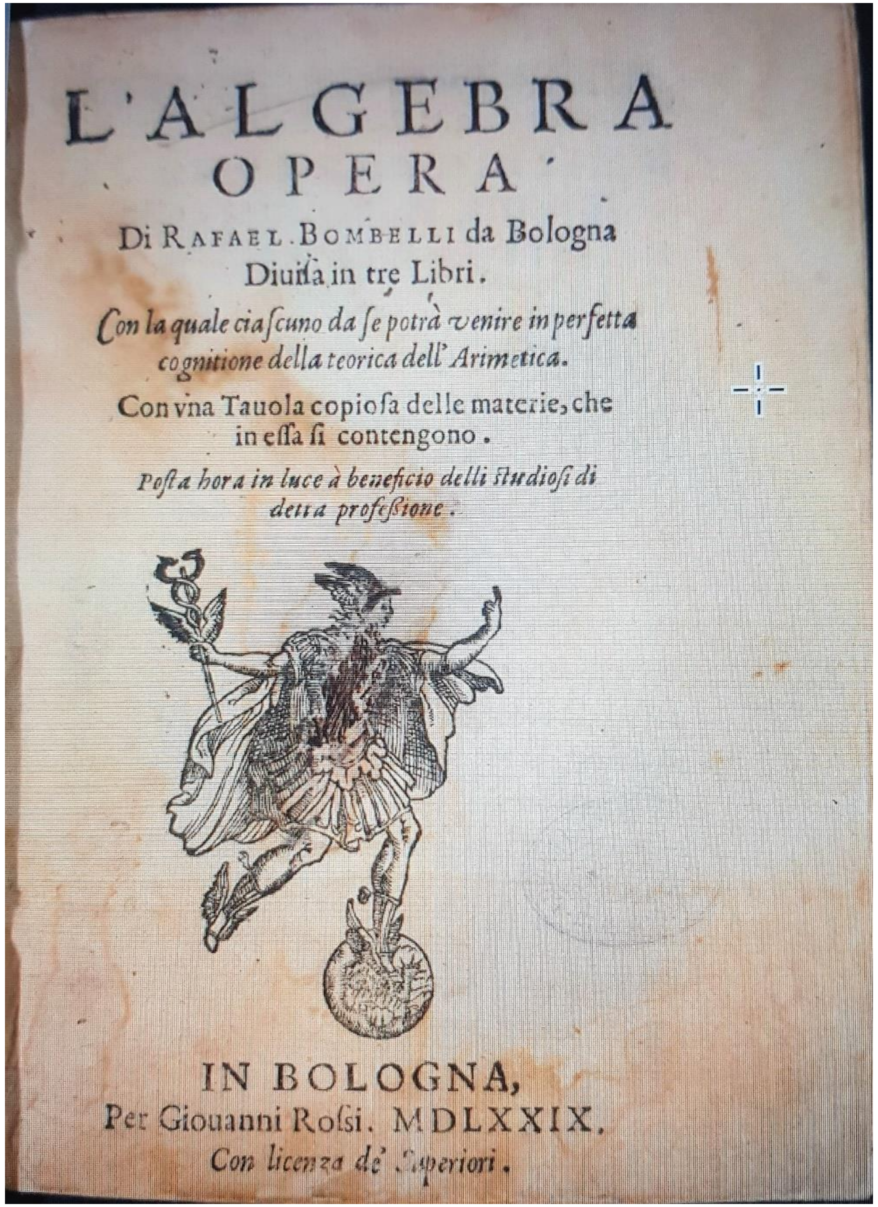
9th European Summer University on the History and Epistemology in Mathematics Education
University of Salerno 18-22 July 2022

In 1572 Rafael Bombelli published his only work, ***L'Algebra*** dedicated to Alessandro Rufini bishop of Melfi, a small city in South of Italy. The work was reprinted in 1579.

It had been written almost twenty years before in five books, but only the first three were published in the XVI century. The last two books were found and published by the mathematician and historian of mathematics of the University of Bologna, Ettore Bortolotti, in 1929.



Guglielmo Libri in his *Histoire des Sciences Mathématiques en Italy* (1840) wrote:

Bombelli's *Algebra* is divided into three books. The first contains the basic elements, calculation with radicals, and with imaginary numbers, the second one contains everything related to the resolution of equations up to the fourth degree, the third is a collection of problems, among which there are some of great difficulty regarding indeterminate analysis.

In this treatise, all the knowledge of algebra of that time is expounded, the proofs are rigorous and complete and the science has a systematic aspect. The notations easily allow us to make calculations and it is well known to what extent notations influenced the development of algebra. The calculation with radicals is fully exposed, as well as the general theory of imaginary quantities, of which the author makes an application to the so-called "irreducible case" ("caso irriducibile").

Bombelli was the first to have generally enunciated the reality of the three roots of an algebraic equation of third degree when all three present themselves in an imaginary form. In many cases he verified this claim by directly extracting the root of the two binomials.

Bombelli's work has contributed in no little way to the progress of mathematics. For the first time we see the rigor of synthesis applied to algebraic proofs.

We will shortly return to Bombelli’s work. For now we may observe that much progress had been made since the algebra of the Arabs was timidly introduced into Latin Europe. Bombelli was well aware of this when he drew a brief history of the development of algebra. He cited the Greek author **Diophantus of Alexandria**, “a certain Maumetto of Mosé, Arab”, (**Muhammad ibn Musa Al-Khwarizmi**), Leonardo Pisano (**Fibonacci**), and lastly “Friar Luca of Borgo San Sepolcro” (**Luca Pacioli**).

Jacopo de Barbari, ritratto di Luca Pacioli

Summa de Arithmetica geometria. Proportioni: et proportionalita:

Continentia de tutta lopera.

De numeri e misure in tutti modi occurrenti.

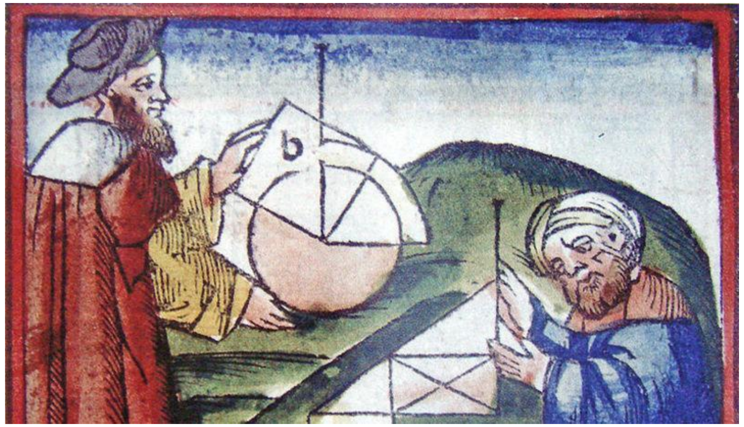
e lor fabriche e fondamenti.

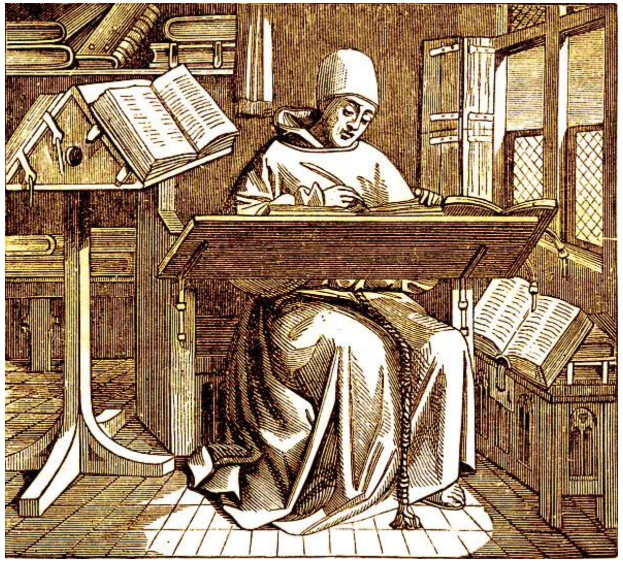
Compagnie in tutti modi: e lor partire.

Socide de bestiami: e lor partire.

L. Pacioli, *Summa de arithmetica, geometria proportioni et proportionalità*, Venezia, 1494

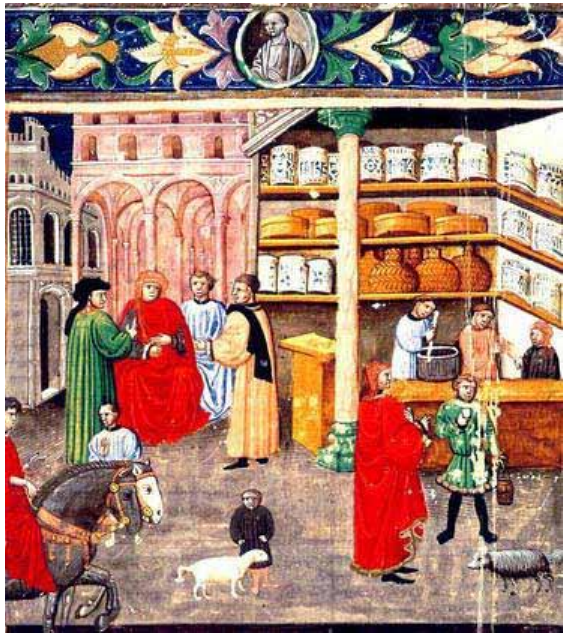
The algebra made its way into Europe “laboriously and slowly”, wrote Carl Boyer, from an Arab tradition transmitted through the Universities, copyists of the ecclesiastical schools, merchant activities, and some scholars from other fields of culture.





Italy was one of the main routes through which the knowledge of the Arabs and algebra, arrived in the West. So, it is not surprising that algebraic research was mainly reborn and developed in Italy .

Gerbert of Aurillac, (pope Sylvester II from 999 since 1003).



The «scuole d’abaco»: an Italian phenomenon

For three centuries, after the *Liber Abaci* of Leonardo Pisano (1202) and before the *Summa* of Pacioli (1494), algebra as well as the Indo-Arabic numbering system, were taught by the Abacus teachers who wrote treatises, which remained unpublished for a long time.

The Abacus schools flourished in the main cities of the North of Italy, like Pisa, Florence, Venice, Brescia, etc. The most famous teachers were called to argue publicly over difficult mathematical problems, in the squares, churches, and in the courts of the princes.

Sometimes, a new theory was developed. This was the case of Leonardo Pisano who solved a problem proposed by Giovanni of Palermo at the court of Frederick II, King of Sicily. He developed the general theory of the “congruous” numbers which is expounded in his treatise *Liber quadratorum*. The problem was to find a square number such that by adding and by subtracting 5, the result is always a square number: $\begin{cases} x^2+5=z^2 \\ x^2-5=y^2 \end{cases}$

A dispute between Abacists and Algorithmists.
Gregor Reisch, *Margarita philosophica*, 1508
The frontespice shows the two arithmetical systems, with the Indo-Arabic digits and with the abacus.

Bologna University: a new teaching for the new arithmetics

From the end of XIV century onwards, a new teaching was introduced, named «Ad lecturam Arithmeticae». In 1496, the teacher was the mathematician **Scipione Del Ferro** who has a place in the history of algebra since he was the first to solve the third degree algebraic equations (how to solve first and second degree equations was already known thanks to the Arab, **Al-Khwarizmi**, *Al-jabr w'al muqabala* the title of his work. **Leonardo Pisano** made the method known in his *Liber Abaci*).

Del Ferro did not publish his method which remained known only to some of his students.

Afterwards, other Italian mathematicians reached the same results and went even further. All the cases of the third and fourth degree algebraic equations were solved, even when in the formula square roots of negative numbers were present.

The protagonists

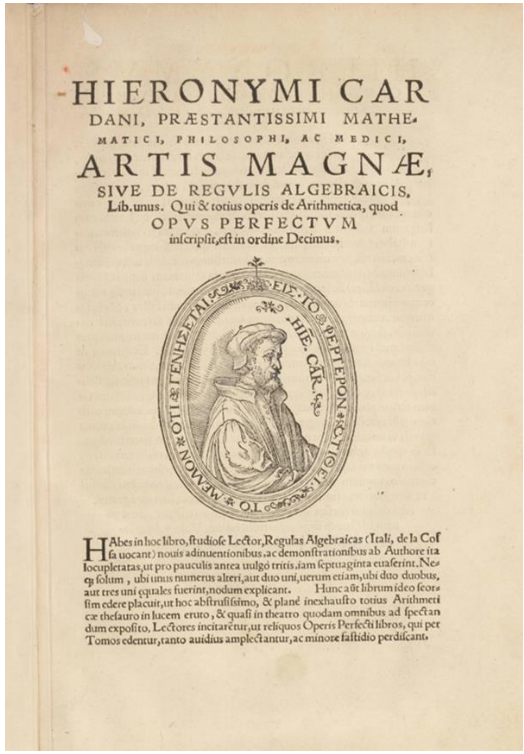
Scipione del Ferro
(Bologna 1465-1526)

Niccolò Tartaglia
(Brescia 1499 circa-1557)

Girolamo Cardano
(Pavia 1501-1576)

Ludovico Ferrari
(Bologna 1522-1565)

Rafael Bombelli
(Bologna 1526-1572 circa)



HIERONYMI CAR
DANI, PRÆSTANTISSIMI MATHE-
MATICI, PHILOSOPHI, AC MEDICI,
ARTIS MAGNÆ,
SIVE DE REGVLIS ALGEBRAICIS,
Lib. unus. Qui & totius operis de Arithmetica, quod
OPVS PERFECTVM
inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cossa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgò tritis, iam septuaginta euaserint. Neq; solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni æquales fuerint, nodum explicant. Hunc aūt librum ideo seorsim edere placuit, ut hoc abstrusissimo, & planè inexhausto totius Arithmeticæ thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarētur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

G. Cardano, *Ars Magna*, 1545 is the first printed work where the third and fourth degree algebraic equations are solved by radicals. Ludovico Ferrari was a scholar of Girolamo Cardano. In the *Ars Magna*, Cardano published a method to solve fourth degree equations attributing the credit to Ferrari. The problem was reduced to solve two other equations, of third and second degree, respectively.

Al-Khwarizmi *Al-jabr w'al-muqabala*

The six kinds of first and second degree algebraic equations and their names

- squares equal to roots means $ax^2 = bx$
- squares equal to number means $ax^2 = c$
- roots equal to number means $ax = c$
- squares and roots equal to number means $ax^2 + bx = c$
- squares and number equal to roots means $ax^2 + c = bx$
- roots and number equal to squares means $bx + c = x^2$

where a, b, c are positive numbers.

The rule

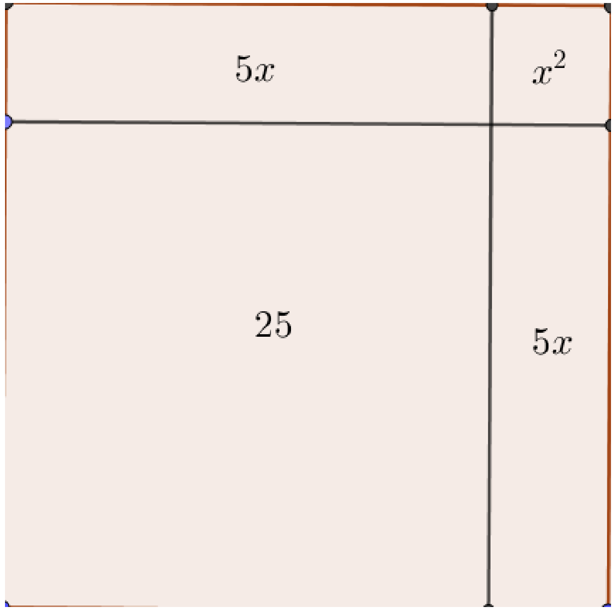
The method to solve the equation $x^2 + 10x = 39$ was taught this way:

«you have to divide into two parts the number of the roots (10), in this case you get 5, then multiply this number by itself (25) and add 39, so you get 64. Now you have to take the square root of 64, that is 8. Subtract from 8 the half of the roots (5), the remainder is 3, the solution of the equation».

This corresponds to the well known formula for the positive solution of the equation $x^2 + px = q$ with p, q positive numbers

$$x = \sqrt{\left(\frac{p}{2}\right)^2 + q} - \frac{p}{2}$$

Completing the square



According to the equation, the two rectangles together with the square x^2 are equal to 39.
The larger square is 39+25= 64
The side of the larger square is $5 + x$ and is also equal to 8
so $x = 8 - 5 = 3$

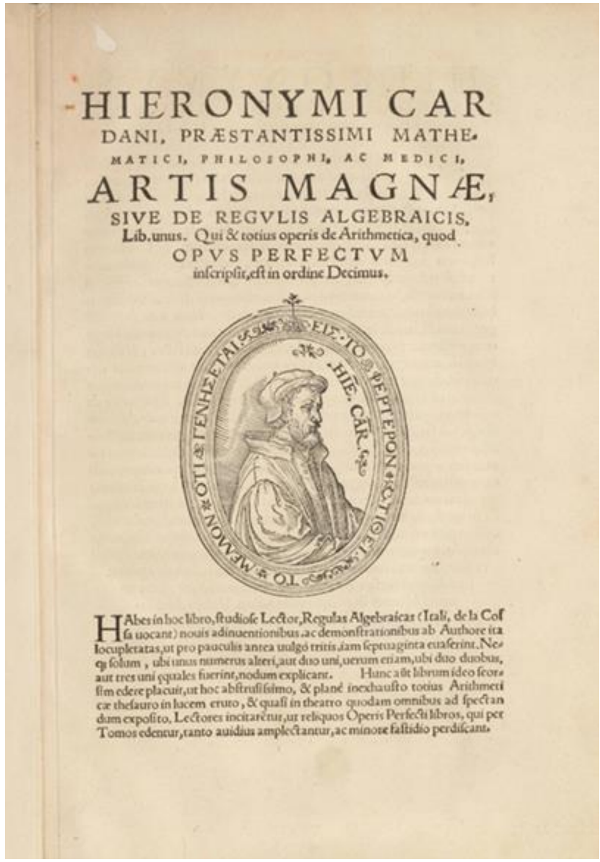
The solution formula for the third degree algebraic equations

As previously mentioned, Cardano published the formula in his ***Ars Magna*** (1545).

He considered three kinds of equations:

- cube and things ("cose") equal number which means $x^3 + ax = b$
- cube and number equal things means $x^3 + b = ax$
- cube equal things and number means $x^3 = ax + b$

with a, b positive numbers. The mathematicians of the XVI century never considered the equation in the form $x^3 + ax + b = 0$, because they interpreted geometrically all the terms of the equations, that is cubes, and rectangular parallelepipeds. Considering only these three kinds of equations was not a lack of generality; it was well known to Cardano that with a change of the unknown, it was possible to eliminate the second degree term (for example, given the equation $x^3 + ax^2 + bx = c$ the change of unknown is $y = x + \frac{a}{3}$).



Tartaglia, Venezia 1534

Quando che'l cubo con le cose appresso
Se agguaglia a qualche numero discreto
Trovan dui altri differenti in esso.
Dapoi terrai questo per consueto
Che'l lor produtto sempre sia eguale
Al terzo cubo delle cose netto.
El residuo poi suo generale
Delli lor lati cubi ben sottratti
Varrà la tua cosa principale.

$$x^3 + px = q$$

$$u^3 - v^3 = q$$

$$u^3 v^3 = \left(\frac{p}{3}\right)^3$$

$$x = u - v$$

In el secondo de codesti atti
Quando che'l cubo restasse lui solo
Tu osserverai quest'altri contratti:
Del numer farai due parti a volo
Che l'una in l'altra si produca schietto
El terzo cubo delle cose in stolo.
Delle qual poi, per comun precetto
Torrai li lati cubi insieme gionti
Et cotal somma sarà il tuo concetto.

El terzo poi di questi nostri conti
Se solve col secondo, se ben guardi
Che per natura son quasi congionti.
Questi trovai, et non con passi tardi,
Nel mille cinquecente quatro e trenta
Con fondamenti ben saldi e gagliardi
Nella città dal mar intorno centa.

$$x^3 = px + q$$

$$u^3 + v^3 = q$$

$$u^3 v^3 = \left(\frac{p}{3}\right)^3$$

$$x = u + v$$

$$x^3 + q = px$$

Completing the cube

$$x^3 + 6x = 20$$

The mathematicians of the XVI century operated only with homogeneous geometrical magnitudes.

x^3 is a cube, so $6x$ is also a solid.

$6x = 3 \times 2x$ is considered as formed by three rectangular parallelepipeds having 2 as a face and x as the relative height. In the figure the three rectangular parallelepipeds have dimensions u, v and $x = u - v$, so

$$\boldsymbol{u \times v = 2} \text{ that is}$$

$$\boldsymbol{u^3 \times v^3 = (\tfrac{6}{3})^3 = 8} \text{ **(Tartaglia second condition)**}$$

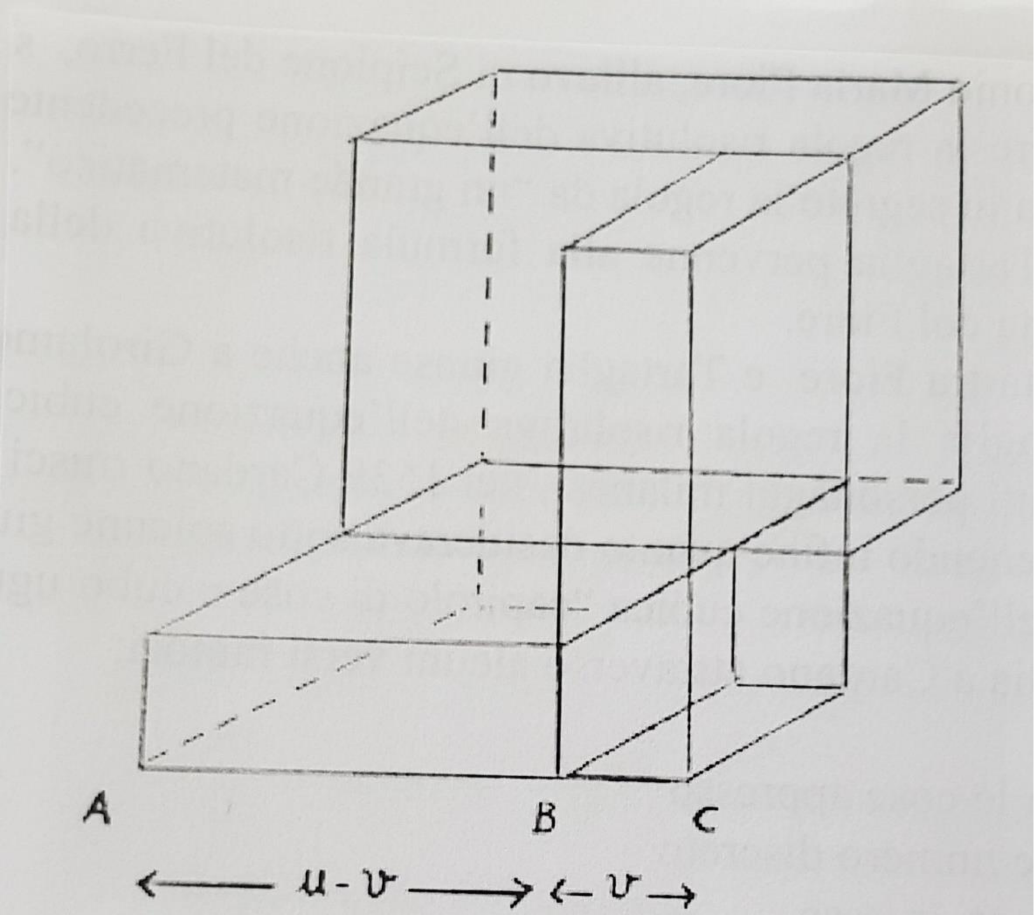
The algebraic expression $x^3 + 6x = \boldsymbol{(u - v)^3 + 3 \times uv\,(u - v)}$ is called "**gnomone**". According to the equation, the "**gnomone**" yields 20.

To complete the cube with edge *AC*=*u*, it is necessary to add to the "**gnomone**", the cube v^3.

$$u^3 = \boldsymbol{(u - v)^3 + 3 \times uv(u - v)} + v^3$$

that is

$$\boldsymbol{u^3 = 20 + v^3} \text{ **(Tartaglia first condition)**}$$



AC= *u*
AB=*x*=*u*-*v*
BC=*v*


To solve the equation $x^3 + 6x = 20$

Let's put into the equation, $x = u - v$. If u and v are solutions of the following system

$$\begin{cases} \boldsymbol{u^3 - v^3 = 20} \\ \boldsymbol{u^3 v^3 = 8} \end{cases}$$

then $x = u - v$ is solution to the equation $x^3 + 6x = 20$.

And the problem is led back to find two numbers u^3 and $-v^3$ of which the sum and the product are known (20 and -8). Such numbers are the solutions to the second degree equation $\boldsymbol{z^2 - 20z - 8 = 0}$, that is $u^3 = 10 + \sqrt{100 + 8}$ and $-v^3 = 10 - \sqrt{100 + 8}$

$$\boldsymbol{x = u - v = \sqrt[3]{\sqrt{108} + 10} - \sqrt[3]{\sqrt{108} - 10}}$$

The rule was claimed for all the equations of this form $x^3 + px = q$. The procedure was the following: let us cube $\frac{p}{3}$ and square $\frac{q}{2}$ and make the sum, $(\frac{p}{3})^3 + (\frac{q}{2})^2$.

Let us take the square root of the latter and add $\frac{q}{2}$ so you have $\sqrt{(\frac{p}{3})^3 + (\frac{q}{2})^2} + \frac{q}{2}$. Now extract the cube root of the latter and subtract the cube root of its «residuo» ($\sqrt{(\frac{p}{3})^3 + (\frac{q}{2})^2} - \frac{q}{2}$) and you will have the solution

$$\boldsymbol{x = \sqrt[3]{\sqrt{(\tfrac{p}{3})^3 + (\tfrac{q}{2})^2} + \tfrac{q}{2}} - \sqrt[3]{\sqrt{(\tfrac{p}{3})^3 + (\tfrac{q}{2})^2} - \tfrac{q}{2}}}$$

The irreducible case of the cubic equation

To solve the equation $x^3 = px + q$, Cardano's method consists in posing

$$x = u + v$$

~~by~~ a procedure like the one followed in the case of the previous kind of equation, the solution formula is:

$$x = u + v = \sqrt[3]{\frac{q}{2} + \sqrt{(\frac{q}{2})^2 - (\frac{p}{3})^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{(\frac{q}{2})^2 - (\frac{p}{3})^3}}$$

The **irreducible case** turns out when $(\frac{p}{3})^3 > (\frac{q}{2})^2$. It was known that the equation admits three real roots, but under the square root a negative number is present. In the *Ars Magna* Cardano carefully avoided this case.

It was the mathematician **Rafael Bombelli** who solved this new and extravagant situation in his work **L'Algebra** (1572).

L'Algebra of Bombelli: the first book

In the first book of *L'Algebra,* Bombelli developed the calculations with square, cubic, fourth, etc. roots. Calculation rules also for the following algebraic expressions:

$\sqrt{a} \pm \sqrt{b}$, $\sqrt{a} \pm b$ (called *binomi* and *recisi*)

$\sqrt{\sqrt{a} \pm \sqrt{b}}$, $\sqrt{\sqrt{a} \pm b}$ (*radici legate*)

$\sqrt[3]{a} \pm \sqrt[3]{b}$ (*binomi cubici*)

$\sqrt[3]{a} \mp \sqrt[3]{ab} + \sqrt[3]{b}$ (*residuo cubico*) (times the corresponding *binomio cubico* you get a rational number)

$\sqrt[3]{\sqrt{a} \pm \sqrt{b}}$, $\sqrt[3]{\sqrt{a} \pm b}$ (*radici cubiche legate*)

He solved the problem «to find the cube side of the *binomio* $\sqrt{n} \pm m$ », that is to transform the expession $\sqrt[3]{\sqrt{n} \pm m}$ into the expression $\sqrt{v} \pm u$

Finally, to solve **the irreducible case** of the third degree equations, he introduced the imaginary numbers and the calculation rules with them.

Bombelli and the arithmetics with the square roots of negative numbers

$\sqrt{-1}$ (the imaginary unit *i*) is called «plus of minus» (*più di meno*)
$-\sqrt{-1}$ (minus the imaginary unit –*i*) is called «minus of minus» (*meno di meno*)

plus times plus of minus is plus of minus	$(+) \times (+i) = +i$
minus times plus of minus is minus of minus	$(-) \times (+i) = -i$
plus times minus of minus is minus of minus	$(+) \times (-i) = -i$
minus times minus of minus is plus of minus	$(-) \times (-i) = +i$
plus of minus times plus of minus is minus	$(+\boldsymbol{i}) \times (+\boldsymbol{i}) = -$
plus of minus times minus of minus is plus	$(+i) \times (-i) = +$
minus of minus times plus of minus is plus	$(-i) \times (+i) = +$
minus times plus of minus is plus of minus	$(-i) \times (-i) = -$

L'Algebra of Bombelli: the second book

Bombelli solved the third and fourth degree algebraic equations including equations which fall into the irreducible case. As an example, he solved the equation

$$x^3 = 15x - 4$$

The Cardano formula contains the «*sofistica*» root $\sqrt{-121}$

$$x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}} = \sqrt[3]{2+11i} + \sqrt[3]{2-11i}$$

It is easy to guess that $x = 4$ is a real root of the equation.

But how to get it from the formula?

In the first book, Bombelli poses the problem «to find the cube side of a *binomio* in which a negative number appears under square root », that is, to transform the expessions $\sqrt[3]{a \pm \sqrt{-b}}$ into the expressions u $\pm \sqrt{-v}$ $(b, v$ positive numbers).

Bombelli is aware that *u* and *v* have to satisfy the two conditions $\begin{cases} a = u^3 - 3uv \\ \sqrt[3]{a^2+b} = u^2 + v \end{cases}$

He proposes a method ***«per pratica»*,** when $a = 2$ and $b = 121$.

In this case $\sqrt[3]{a^2+b} = \sqrt[3]{4+121} = 5$ and so the two conditions are $\begin{cases} 2 = u^3 - 3uv \\ 5 = u^2 + v \end{cases}$

So *u* squared must be less than 5 and *u* cubed must be more than 2. «By experimenting» ***(«a tentoni»)*** Bombelli reaches the conclusion that $u = 2$ and $v = 1$.

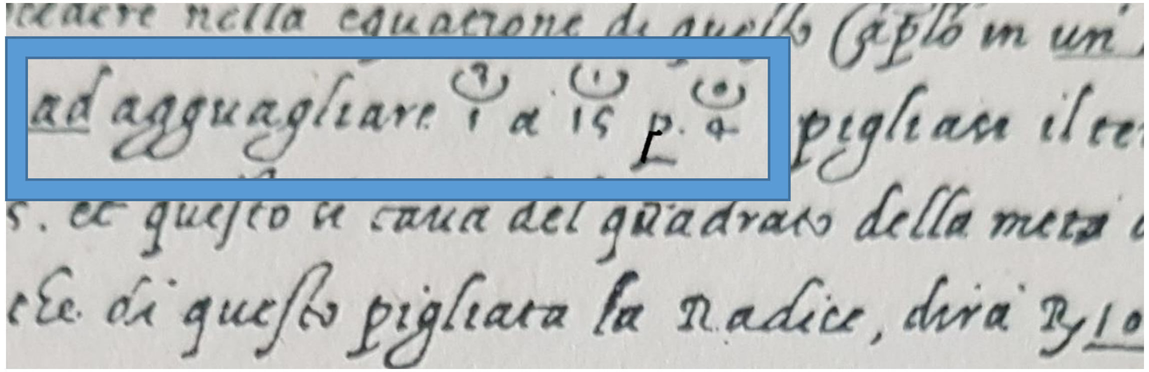
"When the cube roots have side ("lato di numero sano" means positive integer) - claims Bombelli - with this rule, which is not a general rule, but a practical one, it is almost impossible not to find it".

With regard to the solution of the equation $x^3 = 15x - 4$,

$$\sqrt[3]{2+11i} = 2 + i \text{ and } \sqrt[3]{2-11i} = 2 - i$$

and $x = \sqrt[3]{2+11i} + \sqrt[3]{2-11i} = 2 + i + 2 - i = 4$

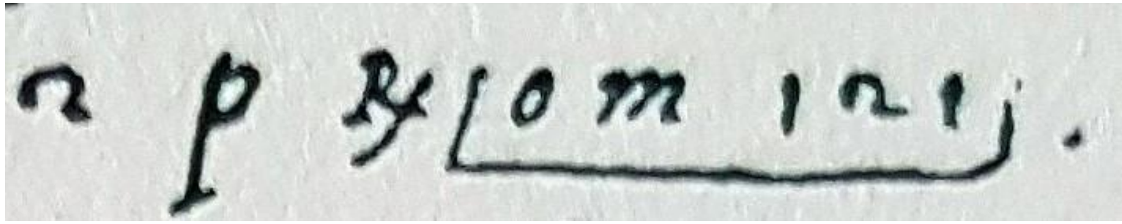
Rafael Bombelli’s algebraic notations



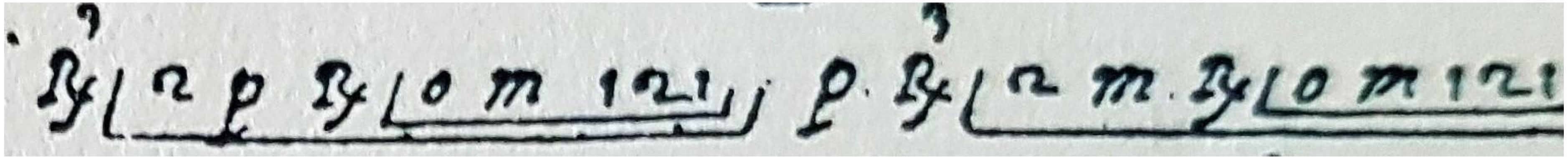
Exponential notation for the powers of the unknown

$$x^3 = 15x + 4$$

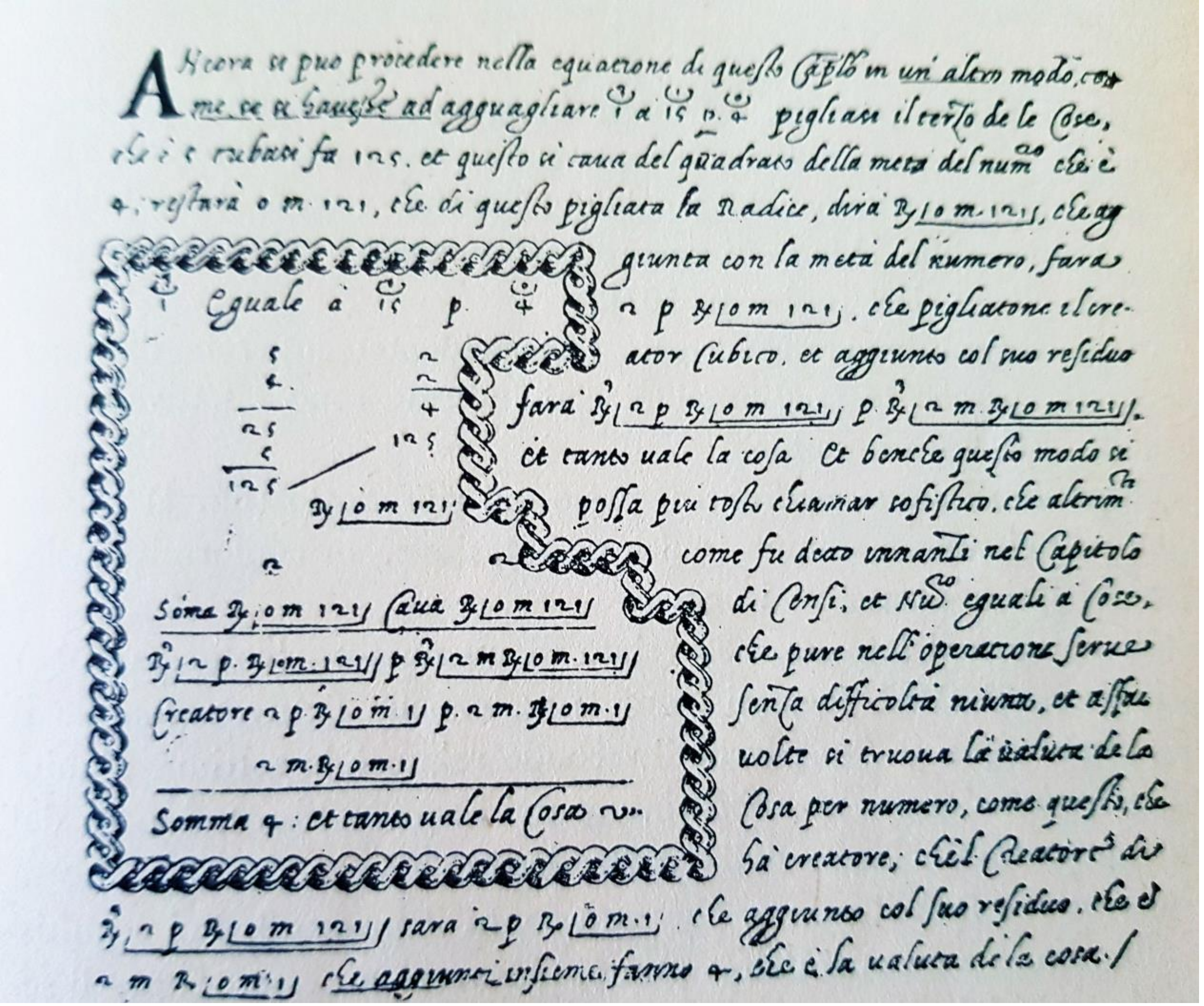
The notation for square and cube roots



$2 + \sqrt{0 - 121}$ that is $2 + 11i$



$\sqrt[3]{2 + \sqrt{0 - 121}} + \sqrt[3]{2 - \sqrt{0 - 121}}$ that is $\sqrt[3]{2 + 11i} + \sqrt[3]{2 - 11i}$



Ancora se puo procedere nella equatione di questo Cap^lo in un' altro modo, come se si havesse ad agguagliare 1 a 15 p. 4 pigliasi il terzo de le Cose, che è 5 cubasi fa 125. et questo si caua del quadrato della meta del num^o che è 4, restarà 0 m. 121, che di questo pigliata la Radice, dirà R|0 m. 121|, che aggiunta con la meta del numero, farà 2 p R|0 m 121|. che pigliatone il creator Cubico, et aggiunto col suo residuo farà R|2 p R|0 m 121|| p. R|2 m. R|0 m 121||. et tanto uale la cosa. Et benche questo modo si possa piu tosto chiamar sofistico, che altrim^te come fu detto innanti nel Capitolo di Censi, et Nu^o eguali a Cose, che pure nell'operatione serue senza difficoltà niuna, et assai uolte si truoua la ualuta de la Cosa per numero, come questo, che hà creatore; che il Creatore di R|2 p R|0 m 121|| sarà 2 p R|0 m. 1| che aggiunto col suo residuo. che è 2 m R|0 m. 1| che aggiunti insieme fanno 4, che è la ualuta de la cosa.

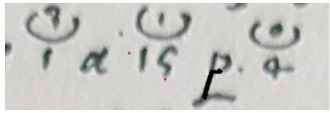
1 Eguale a 15 p. 4

Soma R|0 m 121| Cuba R|0 m 121|

R|2 p. R|0 m. 121|| p R|2 m R|0 m. 121||

Creatore 2 p R|0 m 1| p. 2 m. R|0 m. 1|

Somma 4: et tanto uale la Cosa

Ancora se puo procedere nella equatione di questo Cap[ito]lo in un altro modo, come se si havesse ad agguagliare 

[cioè x^3=15x+4]

pigliasi il terzo de le Cose che è 5, cubasi fa 125 et questo se cava del quadrato della metà del numero che è 4, restarà 0 m 121che di questo pigliasi la Radice, dirà *R*|0 *m* 121| che aggiunta con la metà del numero farà 2 *p* *R*|0 *m* 121| che pigliatone il creatore cubico et aggiunto col suo residuo farà

***R^3* |2 *p R*|0 *m* 121|| *p R^3* |2 *m R*|0 *m* 121||** e tanto vale la cosa.

[cioè $x = \sqrt[3]{2+\sqrt{(0-121}} + \sqrt[3]{2-\sqrt{0-121}}$]

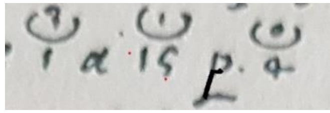
Et benché questo modo se possa più tosto chiamar sofistico che altrim[en]te come fu detto innanzi nel Capitolo di Censi, et Nu[me]ro eguali a Cose, che pure nell'operatione serve senza difficoltà niuna, et assai volte si trova la valuta de la Cosa per numero, come questo che ha creatore, **che il creatore di *R^3* |2 *p R*|0 *m* 121|| sarà 2 *p R*|0 *m* 1|**

[cioè $\sqrt[3]{2+\sqrt{(0-121}} = 2+\sqrt{0-1}$]

che aggiunto col suo residuo che è **2 *m R*|0 *m* 1|**

[cioè $2-\sqrt{0-1}$]

che aggiunti insieme fanno **4** che è la valuta della cosa.

Ancora se puo procedere nella equatione di questo Cap[ito]lo in un altro modo, come se si havesse ad agguagliare 

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***R*^{3} |2 *p R*|0 *m* 121|| *p R*^{3} |2 *m R*|0 *m* 121||** e tanto vale la cosa.

[cioè $x = \sqrt[3]{2 + \sqrt{(0 - 121}} + \sqrt[3]{2 - \sqrt{0 - 121}}$]

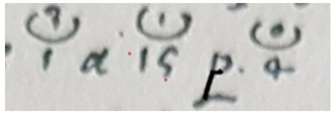
Et benché questo modo se possa più tosto chiamar sofistico che altrim[en]te come fu detto innanzi nel Capitolo di Censi, et Nu[me]ro eguali a Cose, che pure nell'operatione serve senza difficoltà niuna, et assai volte si trova la valuta de la Cosa per numero, come questo che ha creatore, **che il creatore di *R*^{3} |2 *p R*|0 *m* 121|| sarà 2 *p R*|0 *m* 1|**

[cioè $\sqrt[3]{2 + \sqrt{(0 - 121}} = 2 + \sqrt{0 - 1}$]

che aggiunto col suo residuo che è **2 *m R*|0 *m* 1|**

[cioè $2 - \sqrt{0 - 1}$]

che aggiunti insieme fanno **4** che è la valuta della cosa.

Given the equation 

[x^3=15x+4]

Take a third of the «cose» (it means a third of the coefficient of *x* that is of 15, which is 5), make its cube that is 125 and subtract it from the square of the half of the number 4, the difference is 0 m 121. Take the square root of the latter, that is *R*|0 *m* 121| and add the half of the number, 2 *p R*|0 *m* 121|. Take the cube root of it and add the cube root of its «residuo»

***R*^{3} |2 *p R*|0 *m* 121|| *p R*^{3} |2 *m R*|0 *m* 121|| .** This is the unknown *x*.

[$x = \sqrt[3]{2 + \sqrt{(0 - 121}} + \sqrt[3]{2 - \sqrt{0 - 121}}$]

This formula can be called «sofistica» as we said when we considered the equation of second degree [$x^2 + c = bx$] but sometimes you can find the unknown as a number as in this case. The «creatore» of **_R_^{3} |2 *p R*|0 *m* 121||** is **2 *p R*|0 *m* 1|**

[$\sqrt[3]{2 + \sqrt{(0 - 121}} = 2 + \sqrt{0 - 1}$]

If you add it with its «residuo», that is with **2 *m R*|0 *m* 1|**

[$2 - \sqrt{0 - 1}$]

The result is **4,** the value of the unknown.

You can download the *Ars Magna of* Girolamo Cardano and the *L'Algebra* of Rafael Bombelli from some websites:

Internet Archive https://archive.org/

Biblioteca digitale del Museo Galileo https://www.museogalileo.it/it/biblioteca-e-istituto-di-ricerca/biblioteca-digitale/catalogo-biblioteca-digitale.html

In Mathematica Italiana http://mathematica.sns.it/ you can also find biographical news concerning the protagonists of this topic and bibliographical references.