Materials related to the Workshop of ESU-9: *Starting from the history of mathematics in Early Modern Italy: From primary sources to mathematical concepts*. Subsection: *The pre-Newtonian systems of the world*, by Elena Lazzari.

## Worksheet

Surname and name: ________________________________ Class: _____ Date: _______________

# HYPOCYCLOID

***Definition.***

The hypocycloid is the curve described by a fixed point on the circumference of a circle as it rolls on the inside circumference of a fixed circle.

***Property.***

Discover some properties of hypocycloid:

- https://www.geogebra.org/classic/kecuj26k (hypocycloid after one complete revolution inside the base circle)
- https://www.geogebra.org/classic/anrmmtwk (hypocycloid after numerous revolutions inside the base circle)

Vary the sliders $R$ and $r$, which represent the radius of the base circle and epicycle, respectively, and answer the following questions. If the ratio $n$ between the radii $R$ and $r$ is:

- a natural number, the curve is ______________________________________________

  ______________________________________________________________________

- a rational number, the curve is _____________________________________________

  ______________________________________________________________________

Although you cannot experiment with it in GeoGebra, what do you think will happen if $n$ is irrational?

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***Special cases.***

Discover some special cases of the hypocycloid by varying the sliders $R$ and $r$ as required.

- Vary sliders $R$ and $r$ so that $n = 2$. Are you familiar with this curve?

Try with different values of $R$ and $r$ from before, keeping their ratio constant $n = 2$. What differences are there between the new curve and the previous one?

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- Vary sliders $R$ and $r$ so that $n = 4$. Do you know this curve?

  Try with different values of $R$ and $r$ from before, keeping their ratio constant $n = 4$. What differences are there between the new curve and the previous one?

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Try your own experiment, varying the $R$ and $r$ sliders, and look for other interesting curves. Keep track of your best successful attempts.

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# EPICYCLOID

***Definition.***

The epicycloid is the curve described by a fixed point on the circumference of a circle as it rolls on the outside of the circumference of a fixed circle.

***Properties.***

Discover some properties of the epicycloid:

- https://www.geogebra.org/classic/pyryjpja (epicycloid after one complete revolution on the base circle)

- https://www.geogebra.org/classic/mauchabp (epicycloid after numerous revolutions on the base circle)

Vary the sliders $R$ and $r$, which represent the radius of the base circle and epicycle, respectively, and answer the following questions. If the ratio $n$ between the radii $R$ and $r$ is:

- a natural number, the curve is ___

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- a rational number, the curve is ______________________________________________

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Although you cannot experiment with it in GeoGebra, what do you think will happen if $n$ is irrational?

______________________________________________

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***Special cases.***

Discover some special cases of the epicycloid by varying the sliders $R$ and $r$ as required.

- Vary sliders $R$ and $r$ so that $n = 1$. Are you familiar with this curve?

  Try with different values of $R$ and $r$ from before, keeping their ratio constant $n = 1$. What differences are there between the new curve and the previous one?

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- Vary sliders $R$ and $r$ so that $n = 2$. Do you know this curve?

  Try with different values of $R$ and $r$ from before, keeping their ratio constant $n = 2$. What differences are there between the new curve and the previous one?

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Try your own experiment, varying the $R$ and $r$ sliders, and look for other interesting curves. Keep track of your best successful attempts.

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# HYPOTHROCHOID AND EPITHROCHOID

Hypothrocoids and epithrocoids belong to the category of roulette, which are curves generated by one figure rolling on another. When the curves are two circumferences, they are called epitrochoids and hypotrochoids, respectively.

***Definition.***

The hypotrochoid is a roulette traced by a point attached to a circle of radius r rolling around the inside of a base circle of radius R, where the point is a distance d from the center of the interior circle.

The epitrochoid is a roulette traced by a point attached to a circle of radius r rolling around the outside of a fixed circle of radius R, where the point is at a distance d from the center of the exterior circle.

***Special cases.***

Discover some special cases of the epitrochoid by varying the sliders $R$, $r$ and $d$ as required: https://www.geogebra.org/classic/c4w7tdxh.

- Vary the slide $d$ so that $d = 1$. Do you know this curve?

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- Vary the sliders $R$ and $r$ so that $R/r = 1$. Do you know this curve?

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Discover some special cases of the epitrochoid by varying the sliders $R$, $r$ and $d$ as required:

https://www.geogebra.org/classic/zbgjwhuk

- Vary the slide $d$ so that $d = 1$. Do you know this curve?

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- Vary the sliders $R$ and $r$ so that $R/r = 2$. Do you know this curve?

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Try your own experiment, varying the sliders $R$, $r$ and $d$, and look for other interesting curves. Keep track of your best successful attempts.

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<u>**Summary sheet**</u>

# HYPOCYCLOID

***Definition.***

The hypocycloid is the curve described by a fixed point on the circumference of a circle as it rolls on the inside circumference of a fixed circle.

***Property.***

If the ratio *n* between the radii of the base circle and the epicycle is:

- a natural number, the curve is closed and corresponds to an unbraided n-pointed star (trajectory is periodic);
- a rational number, the curve is closed and corresponds to a star (trajectory is periodic);
- an irrational number, the curve is open (trajectory is aperiodic).

***Special cases.***

- A hypocycloid with $n = 2$ is a segment (Cardano's theorem).
- An asteroid is a hypocycloid with $n = 4$.

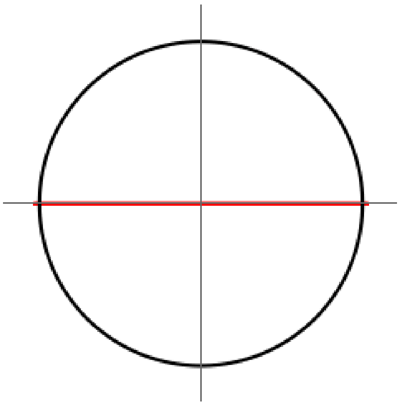

Figure 1. Segment

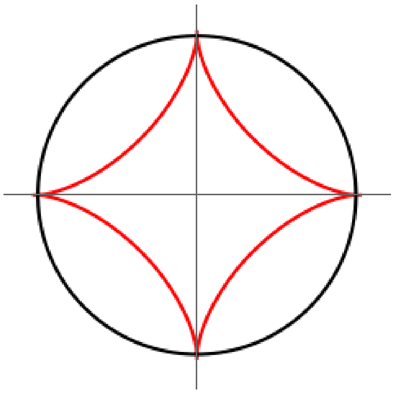

Figure 2. Asteroid

# EPICYCLOID

***Definition.***

The epicycloid is the curve described by a fixed point on the circumference of a circle as it rolls on the outside of the circumference of a fixed circle.

***Properties.***

If the ratio $n$ between the radii of the base circle and the epicycle is:

- a natural number, the curve is closed and unbraided and has n cusps (trajectory is periodic);
- a rational number, the curve is closed and braided (trajectory is periodic);
- an irrational number, the curve is open (trajectory is aperiodic).

***Special cases.***

- A cardioid is an epicycloid with $n = 1$.
- A nephroid is an epicycloid with $n = 2$.

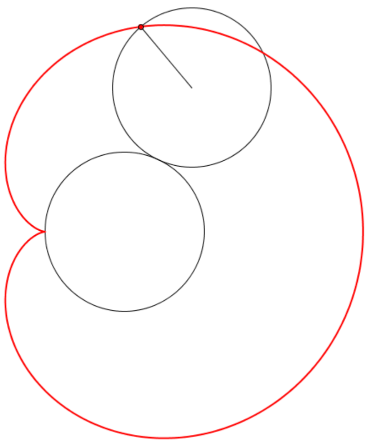

Figure 3. Cardioid

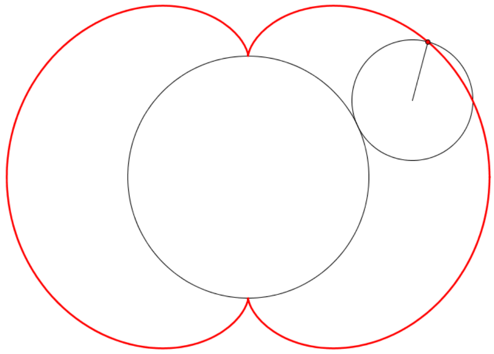

Figure 4. Nephroid

Hypothrocoids and epithrocoids belong to the category of roulette, which are curves generated by one figure rolling on another. When the curves are two circumferences, they are called epitrochoids and hypotrochoids, respectively.

***Definition.***

The hypotrochoid is a roulette traced by a point attached to a circle of radius r rolling around the inside of a base circle of radius R, where the point is a distance d from the center of the interior circle.

The epitrochoid is a roulette traced by a point attached to a circle of radius r rolling around the outside of a fixed circle of radius R, where the point is at a distance d from the center of the exterior circle.

***Special cases.***

- A hypocycloid is a hypothrochoid with $d = r$.
- An ellipse is a hypothrochoid with $R/r = 2$.

- An epicycloid is an epitrochoid with $d = r$.
- A Pascal snail is an epitrochoid with $r = R$.

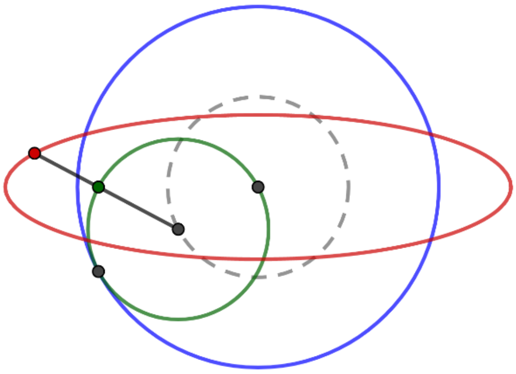

Figura 5. Ellipse

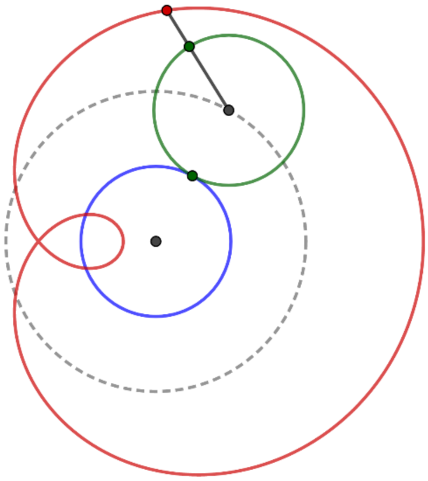

Figura 6. Pascal snail