

Description of the Course (2 CFU- 8 hours)

Quantum groups and their representation theory appeared in theoretical physics in connection to the theory of quantum inverse scattering method, theory which had as aim to construct and solve integrable quantum systems. Independently, Drinfeld and Jimbo showed that the algebras defined by an integrable system are quantum groups (or Hopf algebras), more precisely 1-parameter quantized enveloping algebras of a semisimple Lie algebra \mathfrak{g} (denoted by $U_q\mathfrak{g}$). The further development of quantum group theory uncovered connections of this theory with the quantum theory of angular momentum, 3-dimensional topological field theory (TFT for short), the topology of links, knots and 3-manifolds etc. The notion of quasi-Hopf algebra (or qQG) was introduced by Drinfeld to study categories of modules over $U_q\mathfrak{g}$; by trying to solve this problem he realised that the concept of quantum group is not general enough, and thus one needs generalizations of this concept.

The purpose of this mini course is to present the qQG analog of the Drinfeld and Jimbo quantum groups. Towards this end we will closely follow the plan below.

- we present a theory of 1-cocycles for coalgebras within Yetter-Drinfeld categories;
- we describe the structure of a qQG with a coalgebra projection and relate it to suitable wreath-cowreath structures;
- we provide the invariance of these objects under deformations by 2-cocycles, and in particular by 2-cocycles defined by almost dual pairs in categories of YD-modules;
- we consider the double-biproduct construction for qQG, show that it is isomorphic to a usual biproduct and present its deformations by 2-cocycles;
- define the qQG analogue of $U_q\mathfrak{g}$ as a 2-cocycle deformation of a biproduct and present those quotients of it that lead to various Frobenius-Lusztig quantum kernels type objects.